Intermediaries versus Trolls in Contests for Patents*

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Abstract

Patents are increasingly perceived as ambiguous property rights, as their boundaries are often ill-defined, thereby leading to potential inadvertent infringement and to an explosion in patent litigation. We study the emergence of non-practicing entities and their interaction in the market for patents. While patent trolls monetize their patents through the threat of litigation against alleged infringers, intermediaries instead protect their affiliated firms by buying patents that would otherwise fall in trolls' hands. We develop a model of patent acquisition through a common-value auction incorporating both trolls and intermediaries. We find that firms can never win the auction when individually competing against the troll, and identify conditions under which the seller's revenue increases in response to the troll's participation in the auction. In particular, our results suggest that patent trolls effectively deter producing firms from willfully infringing patents ("efficient infringement") and that sellers of patents that have a mild likelihood of being upheld and found infringed by courts benefit from patent trolls. We then introduce an intermediary who, in exchange for an endogenous membership fee, participates in the auction on firms' behalf by aggregating their bids. While the intermediary's probability to outbid the troll in the auction is positive, his funding mechanism, as a subscription game, greatly hampers his performance in the auction. Our results further suggest that the competition between offensive and defensive NPEs in contests for patents harms the patentholder's revenue. Finally, we elaborate on various policy relevant facets of this complex interaction. JEL Codes: O32, L20, D44, H42

Keywords: Patent assertion entities, Patent infringement, Almost common-value auctions, Forward induction, Asymmetric information

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1 Introduction

In recent years, patent policy has emerged as one of the most active areas of microeconomic policy. One of the main reasons behind this is that innovative knowledge has lost the well-defined property rights it once enjoyed. Patents are often seen as ambiguous and probabilistic property rights, since their boundaries are difficult to determine and often ill-defined[1]. The issue of fuzzy boundaries is especially prominent for patented complex technologies (such as ICT) and business methods[2], thereby increasing the likelihood of inadvertent infringement and ipso facto leading to an explosion of patent litigation (Bessen and Meurer, 2008). The overpropensity to patent together with the preponderance of uncertain patents (see e.g. Lemley and Shapiro (2005), Amir et al., (2014)) have further raised serious doubts among scholars regarding the effectiveness of the current patent system to support innovation (Boldrin and Levine, 2013).

Since holding a patent confers the right to sue alleged infringers to extract monetary compensation, the surge in opportunistic patent monetization has raised public concern and fueled heated policy debates. The last decade has indeed witnessed a growing litigation activity by patent assertion entities (PAEs), also known as patent trolls[3]. Trolls typically do not produce anything covered by their patents, and are therefore frequently referred to as "non-practicing entities" (NPEs). Such firms instead seek to acquire patents so as to use them as a strategic tool to extort rents from alleged infringers[4], through either litigation or the threat of litigation (Scott Morton and Shapiro, 2014). Trolls usually operate in technology fields, such as ICT, where products encompass numerous overlapping patents (Lemley and Melamed, 2013). The likelihood of inadvertently infringing a patented technology is particularly high when R&D intensive firms develop technical components in such complex technological industries where several patented inventions enter their final good (Shapiro, 2001).

Since trolls do not engage in innovative activities, their immunity to countersuits enhances the credibility of their litigation threats (Scott Morton and Shapiro, 2014) and gives them a strategic advantage over producing firms through greater bargaining power when it comes to extracting damage payments from alleged infringers (Fischer and Henkel, 2012), thereby imposing tremendous costs on producing firms and raising concerns regarding their impact on firms' incentives to innovate (Wang, 2010). Bessen et al. (2011) find that over the last four

[1]The lack of well-defined boundaries makes it difficult to assess patent breadth and may thence lead to inadvertent infringement (see e.g. O'Donoghue, 1998).

[2]Patents in these recent areas are more likely to be invalidated and to result in costly litigation. A proposal for a possible remedy is examined in a theoretical model by Levin and Levin (2003).

[3]Lawsuits initiated by PAEs have tripled between 2011 and 2013 (see "Patent Assertion and U.S. Innovation", Executive Office of the President, 2013).

[4]President Obama put it more bluntly: "The folks that you're talking about [PAEs] are a classic example; they don't actually produce anything themselves. They're just trying to essentially leverage and hijack somebody else's idea and see if they can extort some money out of them." (Google+ "Fireside Hangout", 2013).

years, defendants incurred over $80 billion per year in lawsuits initiated by patent trolls. In a similar vein, Bessen and Meurer (2014) estimate that trolls cost society approximately $30 billion per year. For instance, in 2001, NTP Inc., a Virginia-based non-practicing entity, sued Research in Motion (RIM) for infringing on wireless email patents. The court decided in favor of NTP and ordered RIM to pay more than $53 million in damages and to cease infringing the patents. During the appeals process, RIM and NTP negotiated a settlement of their dispute and, in 2006, RIM finally agreed to pay NTP $612.5 million to avoid the injunction.

On the other hand, their proponents instead argue that trolls may support innovation by enabling inventors lacking resources to either manufacture products embedding their technology, license their technology or even enforce their rights, to earn rents. McDonough (2006) argues that patent trolls serve as intermediaries on markets for technology[5] by providing liquidity and market clearing (see also Haber and Werfel, 2016). Likewise, Shrestha (2010) finds that NPEs' patents have significantly higher value than other litigated patents and suggests that NPEs benefit innovation by providing capital to small inventors.

The proliferation of trolls' litigious activity further gave rise to a different type of NPE, often called defensive aggregators, such as RPX Corporation and Allied Security Trust (Hagiu and Yoffie, 2013). Their primary goal is to provide producing firms with safety to operate by acquiring threatening patents that might otherwise fall in the possession of trolls (Wang, 2010). For an annual membership fee, these intermediaries search for patents that might threaten their members with litigation for patent infringement. The identified patents are then collectively financed through members' voluntary contributions. More specifically, each member decides whether to contribute toward the patent purchase, and if so, by how much. Importantly, the contributors' identity as well as the amount they pledge is not disclosed. The intermediary then provides contributors with non-exclusive licenses to the acquired patents, thereby suppressing any risk of patent infringement.

As an attempt to reduce search costs and facilitate patent transactions, large scale patent auctions have been recently developed by patent brokerage companies, attracting operating firms as well as non-practicing entities among bidders[6]. As an example, Ocean Tomo's intellectual property auctions generated about $115 million in transactions between Spring 2006 and Fall 2008 (Hagiu and Yoffie, 2013). NPEs accounted for 61% of the buyers and 82% of the patents sold were bought by NPEs (Caviggioli and Ughetto, 2016).

This paper develops a model of patent acquisition incorporating both trolls and intermediaries, and focuses on the sale of a patent that threatens two producing firms upon enforcement for alleged patent infringement. We focus on the strategic behavior of trolls and producing firms in the patent acquisition process, and study how trolls successfully preempt patents as

[5]See Arora, Gambardella (2010) for a thorough description of markets for technology and Gans, Stern (2010) for challenges hampering their well-functioning.

[6]See for instance http://www.nytimes.com/2009/09/21/technology/21patent.html

compared to firms. We further contribute to the extant literature with the novel incorporation of intermediaries in the model so as to examine their ability to successfully counter trolls' litigious activity by analyzing their collective funding mechanism through firms' individual contributions.

To address these issues, we first consider the sale of a patent through a second-price sealed-bid auction between a troll and two producing firms. We assume that firms use the same technology, but that they operate in different markets so that they are not direct competitors. Prior to the auction, each firm privately receives a signal, which captures her exposure (or likelihood of infringement) to the patent for sale. The value of the patent for a firm equals the damage payments she can extract by asserting it against her rival plus the damages she would have incurred if she were sued for patent infringement. As to the troll, his value for the patent equals the total damages to be obtained by litigating both firms. As such, the patent for sale has common value among bidders. In order to capture the troll's strategic advantage over firms through his immunity to countersuits, it is assumed that he holds a private-value advantage so that he enjoys a strictly higher ex-post valuation for the patent. On the other hand, he is completely uninformed about the common value of the patent.

We show that the troll adopts an extreme equilibrium bidding behavior depending on the magnitude of his private-value advantage, namely, he bids either very aggressively or very cautiously. In turn, firms do not suffer from ex-post regret and may mildly shade their bids down, so that the expected equilibrium revenue of the seller of the patent may increase when the troll participates in the auction. In particular, we find that the presence of a troll effectively deters firms from willfully infringing patents, and that sellers of patents that have a mild likelihood of being upheld and found infringed by courts benefit from patent trolls' activity. Importantly, the troll always wins the auction in any ex-post equilibrium due to his immunity to countersuits, thereby motivating intermediaries' intervention in the market for patents as an attempt to protect producing firms against litigation brought by trolls.

Therefore, we extend the baseline model by introducing an intermediary who, in exchange for a non-refundable up-front membership fee, offers firms to compete against the troll on their behalf in the auction for patent buyout. Upon acceptance of the intermediary's offer, firms then simultaneously choose whether to contribute and if so, the amount of their contribution. The intermediary's bid then simply aggregates firms' contributions. When winning the auction, the intermediary then provides his members with non-exclusive licenses thereby annihilating any risk of litigation for patent infringement.

We show that the intermediary screens out low-signal firms in order to charge a strictly positive membership fee. Moreover, we identify two necessary conditions for the intermediary to outbid the troll and the viability of its business model: both firms must contribute and any excess of contributions must be fully refunded to contributing firms. Nevertheless, because the patent is collectively financed through individual contributions, the collective action issue inherent to the intermediary's funding mechanism greatly hampers his performance in the auction and dramatically lowers the seller's revenue. Indeed, we show that there is no equilibrium in which the intermediary wins the auction at a strictly positive price. We highlight two classes of equilibria yielding two opposite outcomes. The first one exhibits a free rider problem whereby each firm has an incentive to lower her contribution so that the other firm incurs a larger share of the patent purchase. As a result, firms' total contributions are too low and the troll always wins. The second instead involves firms pledging aggressive contributions so that the troll always bids zero to ensure losing. Invoking forward induction arguments (Kohlberg and Mertens, 1986), we argue that the second class of equilibria is more plausible. Thus, the intermediary wins the patent for sale with a strictly positive ex-ante probability, thereby partially overcoming the troll's threat for firms. In sum, our results suggest that the competition between offensive and defensive NPEs harms the patentholder's revenue and may undermine its incentives to innovate in the first place.

Despite the importance of the issue at hand for innovation policy, the existing literature in industrial organization comprises only recent or ongoing work, and focuses on the effect of patent trolls on incentives to undertake R&D and on litigation. Lemus and Temnyalov (2017) examine PAEs' "patent privateering" strategies, which consist of acquiring patents from operating firms to subsequently enforce them against alleged infringers, usually competitors of the patent seller. They analyze the impact of such strategies on incentives to undertake R&D and to engage in costly litigation. Without PAEs, producing firms are reluctant to enforce their patents against their rivals due to the threat of countersuits. In turn, the authors show that outsourcing patent monetization to PAEs enhances the offensive value of patents due to PAEs' immunity to countersuits but undermines their defensive value. In particular, they find that when the former effect prevails, PAEs spur incentives to innovate and enhance social welfare. Hovenkamp (2013) instead develops a dynamic model of patent assertion and reputation building in order to study PAEs' strategy of predatory litigation. By aggressively asserting weak patents against alleged infringers, PAEs develop a tough reputation and gain credibility in their litigation threats, so that other firms are more inclined to settle on a licensing agreement before reaching the courts. While PAEs experience losses when litigating patents that are likely to be invalidated, the author argues that the prospect of substantial licensing payments through subsequent settlement agreements compensates. In a similar vein, Choi and Gerlach (2015) examine PAEs' litigation strategies and the credibility of their threats. They show that naming multiple defendants using related technologies enhances the credibility of their litigation threat and their bargaining position through information externalities generated across litigation suits.

This paper also contributes to the vast literature on auctions. In second-price common-value auctions with two bidders, introducing asymmetries among players through a private-value advantage drastically affects the outcome of the auction, namely, the advantaged bidder always wins and the seller's revenue substantially decreases (see Bikhchandani (1988) and Avery and Kagel (1997)). Levin and Kagel (2005) examine whether this extreme result still obtains with more than one regular bidder in an almost common-value second-price auction where each bidder receives a private signal. They show that, in the wallet auction, regular bidders have a positive probability to win the auction, yet lower than that of the advantaged bidder, and that a small private-value advantage only slightly decreases the seller's revenue. However, while asymmetries in terms of either information or ex-post valuations for the object across bidders have been extensively studied, the existing literature does not incorporate both sources of asymmetries with more than two bidders. As such, the model we consider is therefore at the intersection of these two strands of the literature by considering an almost common-value auction with three asymmetrically informed bidders.

In this respect, our results indicate that perturbing the information structure so that the advantaged bidder is also uninformed restores the extreme result of almost common-value auctions with two imperfectly informed bidders. Namely, the advantaged bidder always wins in any ex-post equilibrium. Furthermore, we identify conditions on the prior distribution of signals so that the participation of an uninformed advantaged bidder substantially raises the seller's expected equilibrium revenue.

The remainder of the paper is organized as follows. Section 2 presents the model and the equilibrium concept. Section 3 characterizes the equilibria of the patent auction in which firms compete against the troll and examines players' exposure to ex-post regret. In Section 4, we extend the model with the introduction of an intermediary who aggregates firms' contributions toward the patent purchase and competes with the troll in the auction on behalf of firms. Concluding remarks are provided in Section 5. Finally, all the proofs are provided in the Appendix.

2 The model

We consider the auction of a patent which, once bought out, might threaten two producing firms (for simplicity) upon enforcement for patent infringement. We assume that firms use the same technology, but that they operate in different markets so that they are not direct competitors. We do not model product market interaction. Rather, we focus on the strategic value of the patent for sale as a weapon to extract rents from alleged infringers[7]. Bidders include the two producing firms (indexed by $i = 1, 2$) and a patent troll (indexed by T), and we denote by $\mathcal{B}$ this set of risk neutral bidders.

[7]Strategic patent acquisition can also enhance the defensive value of its acquirer's patent portfolio in patent infringement countersuits (see e.g. Lemus and Temnyalov, 2017).

2.1 Patent value and information structure

Each firm i is characterized by a different degree of exposure, denoted by $x_i \in [0,1]$, to the patent for sale, which can be thought of as the probability that a court would deem the patent valid and infringed by firm i. Throughout the paper, x_i denotes the realized value of the signal received by firm i. Signals X_1, X_2 are independently and identically drawn from an absolutely continuous distribution F over the support $[0,1]$, with atomless and everywhere strictly positive density $f = F'$, and satisfying the following assumption[8]:

Assumption 1. $\frac{d}{dx}\left(\frac{1-F(x)}{xf(x)}\right) < 0$ *for all* $x \in [0,1]$

We assume that, prior to the auction, each firm privately receives her signal, but remains uninformed about the other firm's degree of exposure. In the sequel, we will also use the following notation: $Y_1 = \underset{i}{max}\{X_i\}$ and $Y_2 = \underset{i}{min}\{X_i\}$.

Acquiring the patent confers its new owner the right to enforce it against potential infringers through litigation, or settlement out of court under the threat of litigation, so as to collect damages $d > 0$. More specifically, the benefits of winning the patent auction for firm i are twofold. First, it allows to save on damages to be paid if the patent is bought out and subsequently enforced by any other bidder. Second, it also entitles firm i to sue the other infringing firm j. As to the troll, the benefit derived from acquiring the patent is to assert it against both firms so as to collect damage fees d. Hence, the value of the patent for sale, v, equals the total expected damages that can be extracted from infringers, that is $v(\boldsymbol{x}) = d.(x_1 + x_2)$, and is common to all bidders. Without loss of generality, we normalize damages d to one so that the common value of the patent reduces to $v(\boldsymbol{x}) = x_1 + x_2$, as in the well-known wallet game (see Klemperer, 1998). That is, firm i's ex-post valuation for the patent is given by[9] $V_i(\boldsymbol{x}) = v(\boldsymbol{x})$ for all $i \in \{1,2\}$.

In contrast to firms, the patent troll is assumed to be completely uninformed[10] about the common value of the patent. However, the troll benefits from a private-value advantage, denoted by λ, so that his ex-post valuation for the patent is

$$V_T(\boldsymbol{x}) = (1+\lambda)v(\boldsymbol{x}) \quad \text{with } \lambda \in [0,1]$$

This common knowledge private-value advantage[11] captures the troll's strategic advantage

[8]As noted by Martimort and Sand-Zantman (2016), a sufficient condition for this assumption to hold is the standard monotone hazard rate condition, i.e., $\frac{d}{dx}\left(\frac{1-F(x)}{f(x)}\right) \leq 0$ (the proof is provided in the appendix).

[9]Following Milgrom and Roberts (1982), firm i's payoff is normalized to zero in the event where she is prosecuted so that her value for the patent equals her opportunity cost of litigation, x_i, plus damage payments, x_j, that she can extract from firm j.

[10]Formally, letting x_T denote the realized signal received by the troll, the common value of the patent is given by $\tilde{v}(x_1, x_2, x_T) = x_1 + x_2 = v(\boldsymbol{x})$.

[11]While we postulate that the private-value advantage enters the troll's ex-post valuation multiplicatively, it may be easily verified that our qualitative results hold if the private-value advantage instead enters additively,

over producing firms, such as his immunity to countersuits for infringement, or his better ability to sue due to a greater expertise in patent assertion activities (see Wang (2010), Scott Morton and Shapiro (2014), Lemus and Temnyalov (2017)).

Once the patent is awarded to the highest bidder, the degree of exposure of the defendant is assumed to be truly revealed to the plaintiff after the latter incurs an information acquisition cost, which we normalize to zero for computational convenience.

The patent is auctioned through a second-price sealed-bid auction[12] with random tie-breaking rule. Thus, the patent is assigned to the highest bidder who pays the second highest bid. We are therefore in the context of a second-price *almost* common-value auction with three asymmetrically informed bidders. In the vast literature on second-price sealed-bid common-value auctions, asymmetries in terms of either information or ex-post valuations for the object across bidders have been extensively studied[13]. Nevertheless, the existing literature does not incorporate both sources of asymmetries with more than two bidders. In this respect, we derive novel results combining insights from both strands of literature.

Let $\boldsymbol{x} = (x_1, x_2) \in [0,1]^2$ denote the vector of signal realizations. Let $\boldsymbol{b}_{-h}$ the vector of bidding strategies of all players but h. The *ex-post* payoff of bidder $h \in \mathcal{B}$ is then given by:

$$u_h(b_h, \boldsymbol{b}_{-h}, \boldsymbol{x}) = [V_h(\boldsymbol{x}) - \max_{l \neq h}\{b_l\}]\mathbb{1}_{b_h \geq \max_{l \neq h}\{b_l\}}$$

where $\mathbb{1}_E$ is the indicator function of event E (i.e., $\mathbb{1}_E$ is equal to one in event E, and zero otherwise).

2.2 Timing and equilibrium concept.

The timing of the game is as follows.

t=0 Signals are simultaneously and independently drawn by Nature from the distribution F over the unit interval, and each firm privately observes her realized signal.

t=1 The patent is auctioned through a second-price sealed-bid auction and the patent reassignee enforces its rights.

It is well known that second-price common-value auctions are plagued by a plethora of equilibria (Milgrom, 1981). Therefore, we first restrict our attention to (pure-strategy) Bayesian

i.e. if the troll's ex-post valuation for the patent is of the form $\tilde{V}_T(x_1, x_2) = v(x_1, x_2) + \kappa$, $\kappa \geq 0$.

[12] We do not endogenize the patent seller's behavior. The seller exogenously sets a reserve price that does not exclude any bidder from participating in the auction.

[13] See for instance Hernando-Veciana (2004) for asymmetries in terms of the information structure; Bikhchandani (1988) and Bulow, Klemperer (2002), Levin, Kagel (2005) for almost common-value auctions with respectively two and strictly more than two bidders. In a similar vein, De Frutos and Pechlivanos (2006) consider a two-bidder model of almost common-value auction with uncertainty regarding whether bidders are advantaged or disadvantaged.

Nash equilibria in undominated strategies (or, undominated equilibria) to eliminate some trivial equilibria that would not be meaningful in our context[14].

For firm 1 (say), bidding $b_1 = v(x_1, 0) = x_1$, i.e. the lowest possible value of the patent given her signal realization, weakly dominates any lower bid. To see this, suppose that firm 1 bids instead according to $b' < x_1$, then the outcome only changes if $b' < \max\{b_2, b_T\} < x_1$. In this case, firm 1 gets

$$u_1(b', \boldsymbol{b}_{-1}, \boldsymbol{x}) = 0 \leq v(\boldsymbol{x}) - x_1 < v(\boldsymbol{x}) - \max\{b_2, b_T\} = u_1(b_1 = x_1, \boldsymbol{b}_{-1}, \boldsymbol{x})$$

Since this holds for all $\boldsymbol{b}_{-1}$, $b_1 = x_1$ weakly dominates any $b' < x_1 = b_1$. A similar argument shows that bidding $b_1 = v(x_1, 1) = x_1 + 1$, i.e. the highest possible value of the patent given her signal realization, weakly dominates any higher bid. Thus, the set of undominated strategies of firm i writes $\mathcal{A}_i(x_i) = [x_i, x_i + 1]$, and an undominated (pure) strategy for firm i is then a function $b_i : [0, 1] \to \mathcal{A}_i(x_i) \subset [0, 2]$ that maps signals into her set of undominated strategies.

Likewise, because the troll is completely uninformed about the common value of the patent, his undominated bids necessarily lie between his lowest possible ex-post valuation (that is, $V_T(\mathbf{0}) = 0$), and his highest possible ex-post valuation (namely, $V_T(\mathbf{1}) = 2(1+\lambda)$), so that his set of undominated strategies is $\mathcal{A}_T = [0, 2(1 + \lambda)]$. Thus, an undominated (pure) strategy for the troll is simply $b_T \in \mathcal{A}_T$.

We can now define the equilibrium notion that we will first use in this paper.

Definition 1. The vector of bid functions $\boldsymbol{b}^* = (b_T^*, b_1^*, b_2^*)$ is an *undominated equilibrium* of the patent auction if for all $h \in \mathcal{B}$, for all $\boldsymbol{x} \in [0, 1]^2$ and all $a_h \in \mathcal{A}_h$,

$$\begin{cases} \mathbb{E}[u_T(\boldsymbol{b}^*(\boldsymbol{X}), \boldsymbol{X})] \geq \mathbb{E}[u_T(a_T, \boldsymbol{b}^*_{-T}(\boldsymbol{X}), \boldsymbol{X})] \\ \mathbb{E}[u_i(\boldsymbol{b}^*(\boldsymbol{X}), \boldsymbol{X})|X_i = x_i] \geq \mathbb{E}[u_i(a_i, \boldsymbol{b}^*_{-i}(X_j), \boldsymbol{X})|X_i = x_i] \quad \forall i \neq j,\ i, j \in \{1, 2\} \end{cases}$$

The first inequality says that bidding b_T^* is optimal for the troll against firms' strategies $\boldsymbol{b}^*_{-T}$, and since he does not hold any information about the common value of the patent, the expectation operator is with respect to the random vector $\boldsymbol{X}$. On the other hand, the second inequality states that bidding b_i^* is optimal for firm i against her competitors' strategies $\boldsymbol{b}^*_{-i}$ when evaluated at the interim stage, that is, once she learns her exposure to the patent.

Another natural refinement in the context of common-value auctions is to further focus on equilibrium strategies satisfying the no ex-post regret property, defined next.

Definition 2. An undominated strategy b_h for bidder $h \in \mathcal{B}$ satisfies the *no ex-post regret property* if for all $\boldsymbol{x} \in [0, 1]^2$ and all $a_h \in \mathcal{A}_h$, $u_h(b_h, \boldsymbol{b}_{-h}(\boldsymbol{x}_{-h}), \boldsymbol{x}) \geq u_h(a_{h,}, \boldsymbol{b}_{-h}(\boldsymbol{x}_{-h}), \boldsymbol{x})$.

[14]For instance, there is a whole class of equilibria in which one player submits a prohibitively high bid, while its competitors bid more conservatively.

In words, a bidder's strategy is immune to ex-post regret if knowing the vector of realized signals $\boldsymbol{x}$ would not induce it to change its bidding behavior, regardless of whether it wins or loses the auction. In the next section, we will see that this desirable property is *always* satisfied by firms' equilibrium strategies when they are symmetric. However, we shall see that this result does not typically carry over to the troll's bidding strategies because of his lack of information about the common value of the patent. Furthermore, his private-value advantage tends to exacerbate his exposure to ex-post regret as it spurs his incentives to bid aggressively.

Finally, we introduce the stronger concept of ex-post (Nash) equilibrium (Crémer and McLean, 1985) generally adopted in common-value auctions, which ensures that the equilibrium vector of bids is immune to ex-post regret for *all* bidders.

Definition 3. The vector of bid functions $\boldsymbol{b}^* = (b_T^*, b_1^*, b_2^*)$ is an *ex-post equilibrium* in undominated strategies of the patent auction if for all $h \in \mathcal{B}$, for all $\boldsymbol{x} \in [0,1]^2$ and all $a_h \in \mathcal{A}_h$,

$$\begin{cases} u_T(\boldsymbol{b}^*(\boldsymbol{x}), \boldsymbol{x}) \geq u_T(a_T, \boldsymbol{b}^*_{-T}(\boldsymbol{x}), \boldsymbol{x}) \\ u_i(\boldsymbol{b}^*(\boldsymbol{x}), \boldsymbol{x}) \geq u_i(a_i, \boldsymbol{b}^*_{-i}(x_j), \boldsymbol{x}) \qquad \forall i \neq j,\ i,j \in \{1,2\} \end{cases}$$

3 Equilibrium analysis

In this section, we restrict our attention to symmetric equilibrium strategies among firms, and show that the troll always bids either zero or aggressively. Then, we analyze the impact of the troll's participation in the auction on the seller's revenue, and finally examine whether the troll suffers from ex-post regret due to the combination of a lack of information and a private-value advantage. Throughout, we assume that firms' symmetric bidding strategies are continuous and strictly increasing in their signal.

3.1 Benchmark: patent acquisition without NPEs

In light of the diverging views among scholars and practioners regarding the impact of PAEs' activity on innovation and welfare, we provide here a short description of alleged infringers' behavior when facing a risk of litigation in the absence of a patent troll, as well as its effect on the patentholder's revenue, so as to later examine the arguments provided by both PAEs' proponents and opponents. We consider the following two scenarios which closely follow from Scott Morton and Shapiro (2014).

Ex-post "good faith" behavior. Under the ex-post "good faith" behavior (henceforth, GFB) scenario, firms are already using the technology covered by the seller's patent when becoming aware of their potential infringement of the patent in question. This may be the result of an incomplete search of prior art, but also of a difficulty to interpret the boundaries of the claims of the patent. We assume here that firms are of “good faith”, whereby they seek to acquire the seller’s patent by participating in the auction so as to annihilate any risk of litigation for patent infringement and to provide the patentholder with monetary compensations.

As noted above, besides suppressing any risk of litigation for patent infringement, acquiring the patent also allows to extract damage payments from other alleged infringers so that firms’ ex-post valuation for the patent is $v(\boldsymbol{x}) = x_1 + x_2$. Thus, we are in the context of a two-bidder *pure* common value auction whenever the GFB scenario prevails in the absence of a patent troll. The following result is well known in the literature on pure common-value auctions and its proof is therefore omitted (see Milgrom (1981), Milgrom and Weber (1982)) .

Lemma 1. *Consider the GFB scenario. Without NPEs, the patent is acquired by the firm facing the highest likelihood of patent infringement. There is a unique symmetric ex-post equilibrium in undominated strategies where firms bid according to* $b^S(x_i) = 2x_i$ *for all* $x_i \in [0, 1]$*,* $i = 1, 2$*.*

Under the GFB scenario, the patentholder (that is, the seller) is therefore *partially* compensated for the infringement of his patent as his ex-post payoff is $2.\min\{x_1, x_2\}$, which is weakly less than the true value of his patent $v(\boldsymbol{x}) = x_1 + x_2$.

Efficient infringement. This practice refers to the *intentional* or *willful* infringement of a patent, whereby a firm deliberately uses a technology infringing on another firm’s patent without monetary compensation. Efficient infringement (henceforth, EI) seems to be more frequent when the patentholder is a small inventor lacking resources to enforce its rights. It may also occur when the firm infringing the patent finds it more profitable to face a risk of patent litigation if she believes that the patent has a significant probability of being invalidated if challenged in court (in which case the infringement case is dismissed). Straightforwardly, under this scenario, the patentholder is not compensated for the infringement of his patent and gets a zero ex-post payoff.

3.2 Symmetric strategies among firms

We now consider the presence of a patent troll on the market for technologies who seeks to acquire patents with a pure offensive motive, that is, to extract damage payments from alleged infringers. Clearly, if firms resort to efficient infringement in his absence, the presence of a patent troll incentivizes them to participate in the patent auction as they now face a credible threat of being sued by the troll if the patent falls in his hands.

If the GFB scenario would instead prevail, the presence of the troll at the patent auction impacts firms’ bidding behavior in two opposite ways. On the one hand, one might expect that the mere participation of the troll in the patent auction will induce firms to bid more cautiously in order to avoid ex-post regret, ceteris paribus. Intuitively, the troll can be thought of as a "noisy bidder" in the sense that the bid he submits does not reflect or contain any relevant information about the patent value. Rather, driving the troll's bidding strategy is the magnitude of his private-value advantage capturing his strategic advantage over firms due to his immunity to countersuits. Given the auction format under consideration, if the troll is the second highest bidder, then the winning firm will likely overpay for the patent and get a negative payoff. In other words, the presence of the troll worsens firms' winner's curse. On the other hand, firms may be incentivized to submit more "aggressive" bids, up to their maximum willingness-to-pay for the patent in the absence of the troll in the auction. By doing so, firms exploit their information advantage over the troll so that the latter strictly prefers to lose the auction for low values of his private-value advantage.

The next result describes the troll's equilibrium bidding behavior in the auction for patent buyout and its consequences on firms' exposure to ex-post regret.

Proposition 1. *The following holds in any undominated equilibrium where firms' bidding strategies are symmetric:*

(i) *The troll bids either his largest or smallest undominated strategy.*

(ii) *Firms do not suffer from ex-post regret regardless of the outcome of the auction.*

Knowing that his participation lowers firms' maximum willingness-to-pay for the patent through a more severe winner's curse, the troll anticipates that firms bid closer to their interim expected value for the patent. It follows that the troll's winner's curse gets milder despite his information disadvantage, which in turn enhances the profitability of winning the patent auction. Because of his lack of information about firms' exposure to the patent, bidding aggressively, that is, above the highest possible value of the patent, ensures that he always wins and that firms do not regret losing since outbidding the troll would result in a strictly negative ex-post payoff upon winning.

We now offer further insight into Proposition 1 by fully characterizing an equilibrium where the troll always preempts the auctioned patent.

Proposition 2. *There exists a continuum of undominated equilibria in which the troll always wins the patent auction. The equilibrium strategies are then*

$$b_T = 2(1+\lambda) \quad \textit{and} \quad b(x_i) \in [x_i, (1+\lambda)(x_i + \mathbb{E}(X_j | X_j \leq x_i))]$$

This equilibrium profile is robust to the troll's private-value advantage vanishing, i.e. as λ goes to zero, which suggests that firms' cautious bidding behavior is mainly driven by the troll's lack of information, yet worsened by the latter's private-value advantage.

Conversely, if firms adopt a more aggressive behavior, we shall see below that the troll then strictly prefers to lose the auction when his private-value advantage is too low as the price to pay upon winning exceeds the true value of the patent. Every firm then infers that, upon winning, the price she has to pay will necessarily be coming from the other firm. Thus, firms behave as if the troll did not participate in the auction. The next result characterizes an equilibrium for which the outcome of the auction depends on the magnitude of the troll's private-value advantage.

Proposition 3. *The following strategies constitute an undominated equilibrium:*

- *if* $\lambda < \hat{\lambda}$*, then* $b_T = 0$ *and* $b(x_i) = 2x_i$
- *if* $\lambda \geq \hat{\lambda}$*, then* $b_T = 2(1+\lambda)$ *and* $b(x_i) \in [x_i, 2x_i]$

with $\hat{\lambda} \equiv \frac{\mathbb{E}(Y_1 - Y_2)}{\mathbb{E}(Y_1 + Y_2)}$.

Low values for the troll's private-value advantage λ refer to the fact that having an immunity to countersuits in patent infringement cases only gives the troll a mild advantage over producing firms. This may occur in technology areas where the issue of overlapping property rights is less severe, in which case firms are less likely to inadvertently infringe on others' patents. In such cases, firms have a strictly positive ex-ante probability of winning the patent auction. Note that the participation of the troll in the auction does not alter firms' equilibrium strategies played under the GFB scenario. Importantly, the mere presence of the troll effectively deters firms from practicing efficient infringement. Regarding the auction literature, this result further suggests that the symmetric equilibrium of the pure common-value auction with two bidders is robust to the introduction of a third uninformed advantaged bidder.

Conversely, in areas where the patent thicket issue is more severe (i.e. for $\lambda \geq \hat{\lambda}$), such as in ICT, semiconductors and biotechnology (Shapiro, 2001), the troll's immunity to countersuits plays a key role. Firms' attempt to prevent the troll from getting the patent is in vain as the troll always wins the auction in any equilibrium. This is consistent with the empirical results of Fischer and Henkel (2012), who find that the likelihood that a patent is acquired by a NPE rather than a producing firm increases in the patent density of its technology field, i.e. the degree to which patents overlap.

Finally, we shall now examine the impact of the presence of a troll on the market for patents on the patentholder's expected revenue. Clearly, if the EI scenario prevails in the absence of a patent troll, then the patentholder strongly benefits from the troll's activity as it incentivizes alleged infringers to participate in the patent acquisition process due to the risk of litigation brought by the patent troll. The patent seller receives positive monetary compensation for the unjust appropriability of its innovation, thereby supporting the view of PAEs' proponents: Patent trolls may help small inventors lacking resources to enforce their rights (e.g. McDonough, 2006). But, as argued by Scott Morton and Shapiro (2014), the impact of patent trolls on incentives to innovate in the first place hinges on several other important factors such as the seller's increase in R&D effort in response to the prospects of damage awards when the patent troll intervenes.

The seller's change in revenue is however not as clear-cut if the GFB scenario instead prevails without the troll. Even though the seller benefits from the addition of a bidder in the auction through harsher competition to acquire the patent for sale, the fact that the troll holds a private-value advantage worsens firms' exposure to the winner's curse, and may incentivize them to submit more cautious bids (Levin and Kagel, 2005). While the latter effect usually dominates the former in almost common-value settings, thereby lowering the seller's revenue, this result does not necessarily carry over here because of information asymmetries across bidders. The troll's lack of information about the common value of the patent together with his private-value advantage encourage him to adopt an extreme bidding behavior. Because he bids either above the highest possible patent value or zero, we previously saw that firms may respond to the troll's participation in the auction by mildly shading their bids down, so that the competitive effect may instead prevail and positively impact the seller's revenue.

The next lemma will prove useful to establish our main result on the impact of the presence of the troll among bidders on the seller's revenue.

Lemma 2. *Let $\bar{\beta}(x) \equiv 2\int_0^x t\frac{f(t)}{F(x)}\,dt$. If the distribution of signals F dominates the standard uniform distribution in terms of reverse hazard rate ($F \underset{r.h.}{\succ} \mathcal{U}$), i.e., $\frac{f(t)}{F(t)} \geq \frac{1}{t}$ for all $t \in [0,1]$, then $\bar{\beta}(x) \geq x$ for all $x \in [0,1]$.*

It is worth noting that, if F dominates the standard uniform distribution in terms of reverse hazard rate, then F first-order stochastically dominates the standard uniform distribution, i.e., $F(x) \leq x$ for all $x \in [0,1]$ (see e.g. Block et al., 1998)[15]. In words, if the distribution of signals satisfies this condition, then firms have a higher probability of having a high degree of exposure to the patent, i.e., the patent for sale is a priori more likely to be upheld and found infringed in court.

Our next result examines the effect of the troll's activity on the seller's expected equilibrium revenue.

Proposition 4. *Suppose that the troll wins the auction in equilibrium and let $b(x_i)$ denote firms' corresponding symmetric equilibrium strategy. The presence of the troll in the market for patents impacts the seller's expected equilibrium revenue as follows:*

1. *If the EI scenario prevails in the absence of the troll, then the seller's expected revenue is always higher.*

[15] Recall that, letting F and G denote two continuous distributions on (say) $[0,1]$, F first-order stochastically dominates G ($F \underset{1}{\succ} G$) if $F(t) \leq G(t)$ for all $t \in [0,1]$.

2. *If the GFB scenario would instead prevail in the absence of the troll, then:*

   (i) *If* $F \underset{r.h.}{\succ} \mathcal{U}$*, then the seller's expected revenue is lower if firms bid strictly below* $\bar{\beta}(x_i)$*, and higher if firms bid above* $\bar{\beta}(x_i)$ *for any* $x_i \in [0,1]$*.*

   (ii) *If* $\mathcal{U} \underset{r.h.}{\succ} F$*, then the seller's expected revenue is always higher.*

Part (i) of this proposition says that sellers of patents which are more likely to be upheld and found infringed in court may lose revenue in response to the participation of a patent troll among bidders if firms shade their bids down below $\bar{\beta}(x_i)$. Intuitively, the patent for sale in this case represents a high threat to operating firms if asserted and has therefore a high offensive value. Firms must therefore bid more aggressively to acquire the patent but are subject to an acute winner's curse due to the presence of the troll in the auction. Hence, whether the seller's revenue increases depends on the severity of firms' response to the winner's curse. In turn, part (ii) of this result can be understood as follows. If $\mathcal{U} \underset{r.h.}{\succ} F$, then the patent has a priori a milder likelihood of being upheld and found infringed by courts. Firms have a lower willingness to pay for such patents since they represent a lower threat and have lesser offensive value when asserted. Thus, the patent troll benefits the seller in this case by increasing the degree of competition among bidders, which in turn increases the patentholder's revenue.

The next example illustrates the variation of the seller's revenue when signals are distributed according to the power function distribution (see Bagnoli and Bergstrom, 2005).

Example. Suppose that signals are distributed according to

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x^\alpha & \text{if } x \in [0,1] \\ 1 & \text{if } x \geq 1 \end{cases} \quad \text{with } \alpha > 0$$

Under the GFB scenario when firms are the sole bidders in the auction, the seller's expected equilibrium revenue is

$$R_0 = 2\mathbb{E}(Y_2) = \frac{4\alpha^2}{(\alpha+1)(2\alpha+1)}$$

If the troll participates in the auction, firms' undominated symmetric bid functions, $b(x_i)$, lie in $[x_i, 2x_i]$ for all x_i. Thus, if the troll wins in equilibrium, then the seller's expected equilibrium revenue is given by

$$R_T \in [\mathbb{E}(Y_1), 2\mathbb{E}(Y_1)] = \left[\frac{2\alpha}{2\alpha+1}, \frac{4\alpha}{2\alpha+1}\right]$$

Straightforward computations show that $\mathbb{E}(Y_1) > 2\mathbb{E}(Y_2)$ for $\alpha \in (0,1)$ so that $\Delta R \equiv R_T -$

$R_0 > 0$ for any $b(x_i) \in [x_i, 2x_i]$. Conversely, we have that $\mathbb{E}(Y_1) < 2\mathbb{E}(Y_2)$ for all $\alpha > 1$. From the proof of Proposition 4, we know that $\mathbb{E}(\bar{\beta}(Y_1)) = 2\mathbb{E}(Y_2)$. Hence, $\Delta R < 0$ if $b(x_i) \in [x_i, \bar{\beta}(x_i))$ and $\Delta R \geq 0$ if $b(x_i) \in [\bar{\beta}(x_i), 2x_i]$. Finally, the reverse hazard rate of F is $\frac{\alpha}{x}$ so that if $\alpha < 1$ (resp. $\alpha > 1$), then $\mathcal{U} \underset{r.h.}{\succ} F$ (resp. $F \underset{r.h.}{\succ} \mathcal{U}$).

Figure 1 illustrates this example by plotting firms' symmetric bidding strategies as a function of the parameter α. The dashed area represents the seller's region of equilibrium revenue when the troll wins the auction (i.e. $[\mathbb{E}(Y_1), 2\mathbb{E}(Y_1)]$), while the red curve is the seller's revenue in the absence of the troll under the GFB scenario (i.e. R_0).

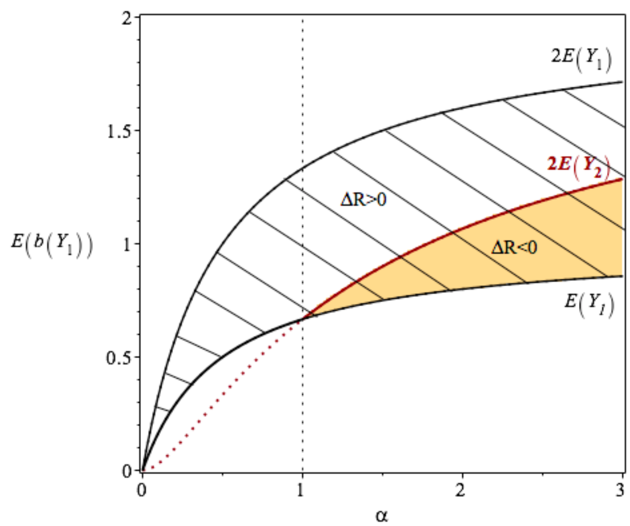


Figure 1: Variation of the seller's expected equilibrium revenue

3.3 On the patent troll's ex-post regret

While firms' symmetric equilibrium bidding strategies are immune to ex-post regret, this property is less likely to be satisfied by the troll's equilibrium bid because of his lack of information about the patent common value. We now restrict attention to equilibrium profiles of strategies satisfying the no ex-post regret property for *all* bidders, and examine whether the set of ex-post equilibria reduces to a unique outcome of the auction.

Before stating the main results, we first illustrate the issue at hand by focusing on the equilibrium profile in which firms play the symmetric strategies of the GFB scenario in the absence of a troll in order to grasp some intuition about the impact of the troll's "all-or-nothing" equilibrium behavior on his exposure to ex-post regret upon both winning and losing the patent. Throughout this subsection, we assume w.l.o.g. that signal realizations are such that $x_1 \geq x_2$.

Lemma 3. *Consider the following equilibrium profile of strategies:*

$$b^S(x_i) = 2x_i \quad \forall x_i \in [0,1], \quad b_T^S = \begin{cases} 2(1+\lambda) & \text{if } \lambda \geq \hat{\lambda} \\ 0 & \text{otherwise} \end{cases} \quad \text{with } \hat{\lambda} \equiv \frac{\mathbb{E}(Y_1 - Y_2)}{\mathbb{E}(Y_1 + Y_2)}$$

Letting $\Delta(\boldsymbol{x}) \equiv \frac{x_1 - x_2}{x_1 + x_2}$*, we have that:*

- *if* $\hat{\lambda} \leq \Delta(\boldsymbol{x})$ *and* $\lambda \in \left[\hat{\lambda}, \Delta(\boldsymbol{x})\right]$*, then the troll suffers from ex-post regret upon winning.*
- *if* $\hat{\lambda} \geq \Delta(\boldsymbol{x})$ *and* $\lambda \in \left[\Delta(\boldsymbol{x}), \hat{\lambda}\right)$*, then the troll suffers from ex-post regret upon losing.*

The idea is that the patent value derived from suing both firms may be relatively low compared to the price paid by the troll for acquiring it, thereby leading to a negative ex-post payoff for the troll unless his private-value advantage is high enough. Conversely, the patent value being higher than expected, the troll could have extracted a positive surplus by winning the patent and asserting it against firms. Finally, note that as $\Delta(\boldsymbol{x})$ goes to the troll's cutoff point, the troll does not suffer from ex-post regret regardless of the outcome of the auction.

Clearly, as the next proposition formalizes, the troll's exposure to ex-post regret upon winning is worsened as firms' ex-ante heterogeneity (or variability) in terms of their exposure to the patent increases, i.e. for distributions of signals that are more spread out.

Proposition 5. *Consider the vector of equilibrium bids* $\boldsymbol{b}^s = (b^s(x_1), b^s(x_2), b_T^s)$ *and let F and G be two continuous distributions with common support* $[0,1]$*. Suppose that the distribution G is a mean-preserving spread of F, i.e.,*

$$\int_0^t [F(t) - G(t)]\, dt \leq 0 \quad \text{for all } t \in [0,1] \quad \text{and} \quad \mathbb{E}_F(X_i) = \mathbb{E}_G(X_i)$$

The troll is less likely to suffer from ex-post regret upon winning (resp. losing) when signals are drawn from F (resp. from G).

Figure 2 provides a partition of the $(\Delta(\boldsymbol{x}), \lambda)$-space showing whether the troll suffers from ex-post regret in equilibrium with the aforementioned vector of bids $\boldsymbol{b}^s$. The troll is ex-post indifferent between winning and losing the auction along the 45° line as

$$\lambda = \frac{x_1 - x_2}{x_1 + x_2} \Leftrightarrow (1+\lambda)(x_1 + x_2) = 2x_1 \Leftrightarrow V_T(\boldsymbol{x}) = b^S(x_1)$$

The upper-half space characterizes all combinations of signal realizations and private-value advantage that yield a strictly positive ex-post payoff to the troll upon winning the patent, while the lower-half space depicts combinations for which the troll strictly prefers to lose the auction from an ex-post perspective.

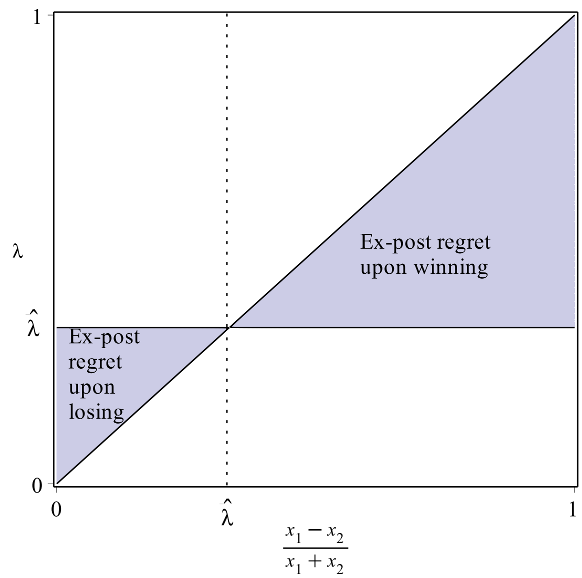


Figure 2: Troll's exposure to ex-post regret

As his private-value advantage goes to zero (resp. to one), the troll is ex-post better off losing (resp. winning) the patent auction for any vector of signal realizations $\boldsymbol{x} \in [0,1]^2$, i.e. for any value of the patent. Instead, the troll is vulnerable to ex-post regret when playing according to $\boldsymbol{b}^S$, i.e. either the upper bound or the lower bound of his set of undominated strategies, for any $\lambda \in (0,1)$. This is due to the fact that he pays the most exposed firm's bid, which does not necessarily capture the full patent value. It follows that the troll likely overpays for the patent, thereby getting a strictly negative ex-post payoff, unless his private-value advantage is sufficiently large to compensate the loss associated with his lack of information. For instance, if firms are very heterogeneous in terms of exposure to patent infringement, i.e. for $x_1 - x_2 \to 1$, then the troll *always* suffers from ex-post regret upon winning since firm 2 faces a very low risk of patent infringement and the troll gets

$$u_T = \lim_{x_2 \to 0} [(1+\lambda)(x_1 + x_2) - 2x_1] = (1+\lambda)x_1 - 2x_1 \leq 0 \quad \forall \lambda \leq 1$$

It is worth noting that this greatly benefits the patent seller as the monetary payments he receives from the troll in this case *exceed* the true value of his patent. Instead, as firms face a similar risk of patent infringement, then the price that the troll would have to pay upon winning tends to the patent *true* value, which in turn would lead to a positive ex-post payoff upon winning as

$$u_T = \lim_{x_2 \to x_1} [(1+\lambda)(x_1+x_2) - 2x_1] = 2\lambda x_1 \geq 0 \quad \forall \lambda \geq 0$$

In such a case, the troll *always* suffers from ex-post regret upon losing, while the seller receives a fair damage payment for the infringement of its patent. In fact, our next result states that, if the troll enjoys a strictly positive private-value advantage, then he must win the patent in any ex-post equilibrium in which firms' bidding functions are symmetric.

Proposition 6. *If $\lambda > 0$, then the troll wins the patent auction in any ex-post equilibrium where firms play symmetric strategies.*

Hence, the set of ex-post equilibria yields a unique outcome, namely, the troll always preempts the patent, which further motivates intermediaries' intervention since firms have no means to protect themselves against threatening patents when individually competing against the troll in the auction.

This result is also of interest with respect to the literature on auctions. In second-price common-value auctions with two bidders, introducing asymmetries among players through a private-value advantage drastically affects the outcome of the auction, namely, the advantaged bidder *always* wins and the seller's revenue substantially decreases as compared to the pure common-value case (see Bikhchandani (1988) and Avery and Kagel (1997))[16]. Levin and Kagel (2005) examine whether this extreme result still obtains with more than one regular bidder in an almost common-value second-price auction where *each* bidder receives a private signal. They show that, in the wallet auction, regular bidders have a positive probability to win the auction, yet lower than that of the advantaged bidder, and that a small private-value advantage only slightly decreases the seller's revenue. However, because of the information structure we adopt, this result does not carry over here since the value-advantaged bidder (namely, the troll) is also uninformed. Indeed, Proposition 6 states that, with more than two players, perturbing the information structure so that the advantaged bidder is also uninformed restores the extreme result of almost common-value auctions with two imperfectly informed bidders: the advantaged bidder always wins.

We now provide a necessary and sufficient condition on the private-value advantage to support the troll's aggressive bidding strategy as part of an ex-post equilibrium in which firms pledge symmetric and linear bids.

Proposition 7. *The strategies $b_T = 2(1+\lambda)$, $b(x_i) = \gamma x_i$ with $\gamma \in [1,2]$, form an ex-post equilibrium in undominated strategies if and only if $\lambda \geq \gamma - 1 \equiv \underline{\lambda}$.*

[16] A notable exception is Umbhauer (2015) who proposes a class of ex-post equilibria in discontinuous strategies for which the advantaged bidder need not always win the auction.

Typically, bidding aggressively makes the troll vulnerable to ex-post regret when firms bid above their signal realization (that is, for $\gamma > 1$). By symmetry and linearity of firms' strategies, the troll pays the bid of the *most* exposed firm upon winning. Yet, the patent value depends on *each* firm's degree of exposure. For instance, if firms are very heterogeneous in terms of exposure to patent infringement, i.e. if $|x_i - x_j|$ is close to one, then the troll will suffer from ex-post regret upon winning, unless his private-value advantage is high enough (namely, such that $\lambda \geq \underline{\lambda}$). In this case, the troll's greater ex-post valuation for the patent due to his immunity to countersuits compensates for his information disadvantage relative to firms when formulating his bid.

In sum, these findings suggest that both the troll's immunity to countersuits and his ability to name multiple defendants in litigation for patent infringement cases play a crucial role in his effectiveness when it comes to acquiring patents that are likely to be infringed.

4 Intermediation in the patent auction

In this section, we extend the previous model by introducing an intermediary who, in exchange of a non-refundable up-front membership fee, enables firms to gather their interests by voluntarily contributing toward the patent purchase (Hagiu and Yoffie, 2013)[17]. The intermediary then aggregates contributions and competes with the troll in the auction for patent buyout on behalf of his members. If winning the auction, the intermediary provides his member(s) with non-exclusive license(s) to the acquired patent, thereby suppressing any risk of litigation for patent infringement brought by the troll.

4.1 Augmented model setup

The crucial difference with the model specified before is that, whenever (say) firm i accepts the intermediary's offer, she then has "private values" for the patent in the sense that she cannot sue firm j for patent infringement, regardless of whether firm j accepted or rejected the offer, $j \neq i$. This comes from the fact that holding a non-exclusive license precludes any right of enforcing the patent, as this right accrues to the patent owner[18]. Therefore, firm i's valuation for the patent now simply equals her degree of exposure x_i, that is, $\tilde{V}_i(x_i) = x_i$, while the troll's valuation remains unchanged due to his ability to enforce the patent against both firms, regardless of whether they joined the intermediary, i.e. $V_T(\boldsymbol{x}) = (1 + \lambda)(x_1 + x_2)$.

[17] See also intermediaries' websites such as: http://www.alliedsecuritytrust.com/Services/AcquisitionModel.aspx and http://www.rpxcorp.com/rpx-services/rpx-defensive-patent-acquisitions/

[18] See Sicom Systems, Ltd v. Agilent Technologies, Inc., 427 F.3d 971, 976 (Fed. Cir. 2005): "A nonexclusive license confers no constitutional standing on the licensee to bring suit or even to join a suit with the patentee because a nonexclusive licensee suffers no legal injury from infringement".

4.1.1 Exclusion of low-signal firms

The intermediary is assumed to be uninformed about firms' signals, or equivalently about the true value of the patent, but makes his offer at the *interim* stage, that is, after each firm privately receives her signal. Straightforwardly, because the intermediary's offer consists of only one contracting variable (i.e. the non-refundable membership fee), he cannot discriminate among firms through signal-contingent membership fees since firms would fail to truthfully self-select within this menu. Put differently, the optimal incentive feasible membership fee bunches signals, so that the intermediary's offer consists of a uniform membership fee, i.e., $t = t_i$ for all $i = 1, 2$, which further implies that proposing a fee targeting the whole set of possible signal realizations, $[0, 1]$, is not profitable since he would then get zero profit[19].

Hence, the intermediary instead chooses a threshold signal $\hat{x} \in (0, 1)$ and a non-refundable membership fee $t > 0$ such that firms with a signal in $[\hat{x}, 1]$ accept his offer, while firms with a signal in $[0, \hat{x})$ reject it. Letting $a_i \in \{A, R\}$ denote the decision of firm i to accept or reject the offer, the intermediary's set of members, $\mathcal{I} \subseteq \{1, 2\}$, is then given by

$$\mathcal{I} = \{i \in \{1, 2\} \text{ such that } a_i = A\} \quad \text{with } |\mathcal{I}| = \sum_{i \in \{1,2\}} \mathbb{1}_{a_i = A}$$

Furthermore, we suppose that the intermediary's number of members becomes common knowledge once firms made their decisions, that is

Assumption 2. $|\mathcal{I}|$ *is common knowledge.*

This assumption comes from the fact that intermediaries' websites often display their number of members, but do not (usually) disclose their identity[20]. In what follows, we let $\Gamma_{|\mathcal{I}|}$ denote the continuation game after $|\mathcal{I}|$ firms accepted the intermediary's offer.

4.1.2 Collective patent purchase through voluntary individual contributions

Upon paying the non-refundable membership fee, firms then simultaneously choose the amount of their contribution, s_i, toward the patent purchase. The intermediary then competes with the troll in the auction for patent buyout where his bid then simply equals the sum of his members' contributions, that is, $b_I = \sum_{i \in \mathcal{I}} s_i$. If $b_I < b_T$, then the intermediary loses the auction and contributions are fully refunded to firms. Instead, upon winning, we leave it open for now whether the intermediary refunds the excess of contributions whenever total contributions exceed the troll's bid (that is, in the case where $b_I = s_i + s_j \geq b_T$).

Observe that the intermediary's funding mechanism for the patent purchase is similar to the well-known *subscription* and *contribution* games in the literature on the private provision

[19]For more details about bunching and shutdown in contract theory, see Laffont and Martimort (2002).

[20]In some cases, they provide the name of some of their members, usually major firms in their technology area. See for instance: http://www.alliedsecuritytrust.com/ASTMembers.aspx

of a discrete public good through voluntary contributions. In such games, agents voluntarily choose the amount of their contribution to the funding of a public good, which is then provided if the sum of contributions exceeds an exogenous threshold cost[21]. Following the terminology of Admati and Perry (1991), contributions are fully refunded in *subscription* games whenever insufficient to provide the public good, as opposed to *contribution* games in which they are retained by the collector. However, in our model, the threshold level is *endogenously* determined by the troll's bidding strategy given the auction format.

Finally, intermediaries publicly commit to not litigate non-members in order to extract revenues

> "Unlike a troll, we do not opportunistically license our patents [...]. And we never offensively assert or litigate the patents we own. RPX provides a purely defensive service. Our goal is to acquire and clear potentially problematic patents so that patent trolls cannot assert them against our clients."[22]

Consequently, we assume that if only one firm accepts, then the intermediary does not sue the non-member firm whenever he wins the patent.

4.1.3 Payoffs

Suppose first that at least one firm decided to join the intermediary. For a given pair of threshold signal and membership fee $(\hat{x}, t)$, let $\boldsymbol{\sigma} = (\boldsymbol{s}, b_T)$ denote the vector of actions in the continuation game $\Gamma_{|\mathcal{I}|}$, where $\boldsymbol{s}$ is the $|\mathcal{I}|$- dimensional vector of contributions and b_T is the troll's bid. Since the intermediary's number of members is common knowledge, firm i can then infer firm j's acceptance decision. The ex-post *net* payoff of firm i with signal x_i, when choosing a_i and contributing s_i, is thus given by $\tilde{u}_i(s_i, \boldsymbol{\sigma}_{-i}, \boldsymbol{x}|(a_i, a_j))$. More specifically, if both firms accept, then firm i, $i = 1, 2$, gets

$$\tilde{u}_i(s_i, \boldsymbol{\sigma}_{-i}, \boldsymbol{x}|(A, A)) = \begin{cases} x_i - s_i - t & \text{if } b_I = s_i + s_j \geq b_T \\ -t & \text{otherwise} \end{cases}$$

Following our above discussion about intermediaries' commitment to not litigate, if firm i accepts and firm j rejects, $i \neq j$, firms' ex-post payoffs are then given by

$$\tilde{u}_i(s_i, \boldsymbol{\sigma}_{-i}, \boldsymbol{x}|(A, R)) = \begin{cases} x_i - s_i - t & \text{if } b_I = s_i \geq b_T \\ -t & \text{otherwise} \end{cases}$$

[21]See for instance Menezes et al. (2001).
[22]Retrieved from: http://www.rpxcorp.com/network/why-join-rpx/

and

$$\tilde{u}_j(s_j, \boldsymbol{\sigma}_{-j}, \boldsymbol{x}|(R,A)) = \begin{cases} x_j & \text{if } b_I = s_i \geq b_T \\ 0 & \text{otherwise} \end{cases}$$

If neither firm accepts the intermediary's offer, then firms' ex-post payoffs are the same as those of the patent auction without the intermediary (see subsection 2.1), namely,

$$\tilde{u}_i(s_i, \boldsymbol{\sigma}_{-i}, \boldsymbol{x}|(R,R)) = u_i(b_i, \boldsymbol{b}_{-i}(x_j), \boldsymbol{x}) = [V_i(\boldsymbol{x}) - \max_{l \neq i}\{b_l\}]\mathbb{1}_{b_i \geq \max\limits_{l \neq i}\{b_l\}} \quad \forall i = 1, 2$$

Finally, upon observing $|\mathcal{I}|$, the troll's ex-post payoff is

$$\tilde{u}_T(b_T, \boldsymbol{\sigma}_{-T}, \boldsymbol{x}||\mathcal{I}|) = \begin{cases} u_T(b_T, \boldsymbol{b}_{-T}(\boldsymbol{x}), \boldsymbol{x}) = [V_T(\boldsymbol{x}) - \max_{l \neq T}\{b_l\}]\mathbb{1}_{b_T \geq \max\limits_{l \neq T}\{b_l\}} & \text{if } \quad \mathcal{I} = \emptyset \\ [V_T(\boldsymbol{x}) - b_I]\, \mathbb{1}_{b_T \geq b_I} & \text{otherwise} \end{cases}$$

4.1.4 Timing and equilibrium concept

The game now unfolds as follows.

t=0 Signals are simultaneously and independently drawn by Nature from the distribution F over the unit interval, and each firm privately observes her realized signal.

t=1 The intermediary chooses a threshold signal and proposes a non-refundable membership fee, and firms simultaneously either accept or reject the intermediary's offer[23].

If both firms reject ($\mathcal{I} = \emptyset$), then the game of Section 2 is played. If instead at least one firm accepts ($\mathcal{I} \neq \emptyset$), then the game proceeds as follows:

t=2 Each member simultaneously submits a voluntary contribution toward the patent purchase.

t=3 The patent is auctioned through a second-price sealed-bid auction between the intermediary and the troll, and:

- if the troll wins, then he enforces his rights,
- if the intermediary wins, then he provides his member(s) with non-exclusive licenses to the acquired patent.

A pure strategy for firm i is now a pair (a_i, s_i) where $a_i \in \{A, R\}$ denotes firm i's decision to accept or reject the intermediary's offer, and $s_i : [0,1] \to \mathbb{R}_+$ is the contribution of firm i to the patent purchase, $i = 1, 2$. Since the intermediary's bid is simply the sum of his members'

[23]Throughout, we adopt the conventional assumption that, when indifferent, firms accept the intermediary's offer.

contributions, he can therefore be thought of as a "passive" bidder in the auction. Thus, given that firms simultaneously pledge their contributions, together with the fact that they are not observable by the troll, the continuation game $\Gamma_{|\mathcal{I}|}$ can then be treated as a one-shot game.

Throughout, we will use the concept of perfect Bayesian equilibrium with the additional requirement that any candidate vector of actions $\boldsymbol{\sigma}^*$ form an ex-post equilibrium of the corresponding continuation game.

Definition 4. The vector of actions $\boldsymbol{\sigma}^* = (\boldsymbol{s}^*, b_T^*)$ is an ex-post equilibrium of the continuation game $\Gamma_{|\mathcal{I}|}$ if for all $\boldsymbol{x} \in [0,1]^2$,

$$\begin{cases} \tilde{u}_T(\boldsymbol{\sigma}^*(\boldsymbol{x}), \boldsymbol{x} || \mathcal{I}|) \geq \tilde{u}_T(b'_T, \boldsymbol{\sigma}^*_{-T}(\boldsymbol{x}), \boldsymbol{x} || \mathcal{I}|) & \forall b'_T \in \mathbb{R}_+ \\ \tilde{u}_i(\boldsymbol{\sigma}^*(\boldsymbol{x}), \boldsymbol{x} | (a_i, a_j)) \geq \tilde{u}_i(s'_i, \boldsymbol{\sigma}^*_{-i}(x_j), \boldsymbol{x} | (a_i, a_j)) & \forall s'_i \in \mathbb{R}_+, \ \forall i \neq j, \ i, j = 1, 2 \end{cases}$$

Definition 5. An equilibrium of the intermediated patent auction is a threshold signal $\hat{x}^*$ and a membership fee t^* such that $(\hat{x}^*, t^*)$ is optimal for the intermediary given other players' strategies, together with a vector of decisions $\boldsymbol{a}^* = (a_1^*, a_2^*)$ satisfying

$$\begin{cases} a_i^* = A & \Rightarrow x_i \in [\hat{x}^*, 1] \\ a_i^* = R & \Leftarrow x_i \in [0, \hat{x}^*) \end{cases} \quad \text{for all } i = 1, 2$$

and such that:

1. Letting $S_i \subseteq [0,1]$ and $S_j \subseteq [0,1]$, the troll and firms' updated beliefs are compatible with Bayes' rule:

$$\begin{cases} \mu_T \hat{=} \Pr\left(\boldsymbol{X} \in S_i \times S_j | \, |\mathcal{I}| = k\right) & = \frac{\Pr(|\mathcal{I}|=k | \boldsymbol{X} \in S_i \times S_j) \Pr(\boldsymbol{X} \in S_i \times S_j)}{\Pr(|\mathcal{I}|=k)} \\ \mu_i \hat{=} \Pr\left(X_j \in S_j | \, |\mathcal{I}| = k, X_i = x_i\right) & = \frac{\Pr(|\mathcal{I}|=k, X_i = x_i | X_j \in S_j) \Pr(X_j \in S_j)}{\Pr(|\mathcal{I}|=k)} \end{cases} \quad \forall i, j = 1, 2 \quad i \neq j$$

2. The vector of actions $\boldsymbol{\sigma}^* = (\boldsymbol{s}^*, b_T^*)$ satisfies Definition 4 given the system of beliefs $\boldsymbol{\mu}$, the intermediary's strategy $(\hat{x}^*, t^*)$ and the vector of firms' decisions $\boldsymbol{a}^*$.

4.2 Equilibria of the continuation games

In this subsection, we examine, for a fixed threshold $\hat{x} \in (0,1)$ and membership fee $t > 0$, firms' strategic behavior when contributing toward the patent purchase, and characterize equilibria of the continuation games that begin after either one firm or both of them accepted the intermediary's offer.

Remark. If neither firm accepts the intermediary's offer (that is, if $\mathcal{I} = \emptyset$), then the troll's set of undominated strategies shrinks as he infers that the *true* patent value satisfies $v(\boldsymbol{x}) \leq 2\hat{x}$.

In particular, standard arguments show that bidding $b_T = 2(1+\lambda)\hat{x}$ weakly dominates any higher bid so that the troll's set of undominated strategies becomes $[0, 2(1+\lambda)\hat{x}] \subset \mathcal{A}_T$ for any threshold $\hat{x} \in (0,1)$. While the troll bids less aggressively than before, it is easy to see that the results of the baseline model without intermediation do not qualitatively change. Namely, the troll wins the auction in any ex-post equilibrium of the continuation game Γ_0 (cf. Proposition 6).

4.2.1 The case of a single member firm

Consider the continuation game after only one firm accepted the intermediary's offer. Since non-exclusive licensees *cannot* sue for patent infringement, this firm has now private values for the patent upon being the only member, so that asymmetries across bidders are exacerbated. Besides his private-value advantage λ, the troll now also benefits from a greater ex-post valuation for the patent relative to firms coming from his ability to name multiple defendants when litigating for patent infringement[24]. Because the intermediary's funding mechanism for the patent purchase relies exclusively on his members' contributions, it follows that the troll *always* preempts the patent for sale whenever only one firm contributes. To see this, suppose that, say, firm 1 accepted the offer while firm 2 rejected it, so that the intermediary's bid is simply equal to the contribution of firm 1. Routine arguments show that firm 1 has a unique weakly dominant strategy, to contribute her true value for the patent, i.e. $s(x_1) = x_1$. In turn, the troll's optimal strategy is then to submit a bid that ensures winning with probability one since his ex-post payoff upon winning is then $(1+\lambda)(x_1+x_2) - x_1 > 0$. Thus, as the following proposition formalizes, the intermediary cannot preempt the patent for sale with only one contributor. In fact, since the membership fee is non-refundable, firm 1 is instead strictly worse off with the intermediary's intervention (relative to section 3) whenever she ends up being the only contributing firm.

Proposition 8. *The intermediary always loses the patent auction in any ex-post equilibrium if only one firm accepts his offer. There is a continuum of ex-post equilibria with* $s(x_1) = x_1$ *and* $b_T \in (1, (1+\lambda)(1+\hat{x})]$.

An insufficient number of contributors to finance the patent for sale is a major impediment in the performance of the intermediary. The troll always wins the patent in equilibrium and neither player suffers from ex-post regret since the intermediary's bid, $b_I = s(x_1) = x_1$, is lower than the patent true value, $v(\boldsymbol{x}) = x_1 + x_2$. This result is robust as the troll's private-value advantage vanishes (that is, for $\lambda = 0$), so that he *always* outbids the intermediary and wins the patent regardless of the magnitude of his private-value advantage λ. Driving this result is the fact that the troll has a higher ex-post valuation for the patent as compared to

[24] PAEs' tendency to name multiple defendants in patent infringement lawsuits allows them to benefit from scale economies by avoiding the duplication of litigation costs (PwC Litigation Study, 2013).

firm 1 since he can also extract damage payments from firm 2 upon winning, while firm 1 now has "private values" for the patent due to her inability to enforce the patent when holding a non-exclusive license. Therefore, the troll's ability to extract the whole value of the patent for sale through litigation (or the threat of litigation) compensates for his lack of information about firms' likelihood of infringement.

Importantly, observe that if winning the patent auction were feasible with only one contributor, firms' incentives to join the intermediary would be greatly undermined. Indeed, by rejecting the intermediary's offer, a firm could then be *de facto* protected from litigation brought by the troll while saving on the membership fee if the other firm instead accepted the intermediary's offer. Clearly, the intermediary would then have to offer a lower subscription fee in order to account for such incentives to free ride. Hence, this result also ensures that firms' incentives to accept the intermediary's offer in the first place are preserved.

4.2.2 Collective action issue with multiple contributing firms

We now turn to the case where both firms accepted the intermediary's offer. From Proposition 8, a necessary condition for the intermediary to outbid the troll in the patent auction is that both firms accept his offer. Nevertheless, we now shall see that this is not a sufficient condition for his success in the auction.

While the fact that the intermediary cannot acquire the patent for sale with only one contributor suppresses firms' incentives to free ride when deciding whether to accept his offer, his mechanism to finance the patent purchase creates a collective action problem among its member firms which potentially undermines his performance in the auction. Because the patent is collectively financed through voluntary individual contributions, this creates a free-rider problem whereby a firm has an incentive to slightly lower her contribution so that the other contributing firm incurs a larger share of the patent purchase, ceteris paribus. This is due to the fact that, whenever both firms join the intermediary, the patent becomes a *collective good* in the sense that both firms (i) benefit from its acquisition by the intermediary through the non-exclusive license they receive, and (ii) are entitled to get the license regardless of their contribution (Olson, 1965).

Proposition 9. *Suppose that both firms join the intermediary. There is an ex-post equilibrium in which the intermediary wins the auction only if the excess of contributions is refunded.*

In the sequel, we shall assume that, upon winning, the intermediary uses a standard *proportional* rebate rule[25] whenever the total contributions exceed the troll's bid. Namely, if

[25]See Spencer et al. (2009) for alternative rebate rules in the context of public good provision.

$b_I = s_i + s_j \geq b_T$, firm i then retrieves

$$r_i(s_i, s_j, b_T) = \frac{s_i}{s_i + s_j}\,(s_i + s_j - b_T)$$

The next result sheds light on the strong impact of free riding on the outcome of the auction.

Proposition 10. *Assume firms pledge symmetric contributions. There is no ex-post equilibrium in which the troll submits a strictly positive bid and the intermediary wins the auction.*

The collective action issue inherent to the intermediary's funding mechanism greatly benefits the troll, as submitting a strictly positive bid triggers firms' free-riding behavior and undermines the intermediary's bid, thereby increasing the profitability of winning the auction for the troll, despite his lack of information about the patent value.

In what follows, we focus on equilibria involving symmetric linear contributions of the form $s(x_i) = kx_i$, $k \geq 0$, so that the intermediary's bid amounts to $b_I(\boldsymbol{x}) = k(x_1 + x_2) \equiv kv(\boldsymbol{x})$. Importantly, beyond their tractability, linear contributions make the troll immune to ex-post regret regardless of whether he wins or loses the auction since the intermediary's bid aggregates firms' private information about the patent true value. To begin with, we propose a class of ex-post equilibria which illustrates the collective action issue at hand. Formally, any profile of strategies of the form

$$\sigma^w = \{s^w(x_i) = \underline{k}x_i \text{ with } 0 \leq \underline{k} < 1 + \lambda,\ b_T^w = 2(1 + \lambda)\}$$

constitutes an ex-post equilibrium of the continuation game Γ_2, in which the intermediary loses the patent auction for sure. To see this, suppose first that firms play according to σ^w. By bidding $b_T^w = 2(1 + \lambda)$, the troll wins the auction for sure as

$$b_T^w = 2(1 + \lambda) \geq (1 + \lambda)(x_1 + x_2) > \underline{k}(x_1 + x_2) = b_I$$

and gets $(1 + \lambda)(x_1 + x_2) - \underline{k}(x_1 + x_2) > 0$ for any $\underline{k} < 1 + \lambda$ which ensures that he does not regret winning. Given the auction format, bidding any $\bar{b} > b_T^w$ does not improve his payoff, while bidding according to $\underline{b} < b_T^w$ triggers a positive probability to lose the auction if $\underline{b} < b_I$, then resulting in a zero ex-post payoff. Similarly, consider, say, firm i and suppose that the troll and firm $j \neq i$ play according to σ^w. Contributing according to $s^w(x_i) = \underline{k}x_i$ leads to the intermediary's defeat in the auction, with associated ex-post payoff $-t < 0$. Obviously, any lower contribution yields the same auction outcome and ex-post payoff. Pledging instead any $\bar{s} > s^w(x_i) = \underline{k}x_i$ changes the outcome only if $\bar{s} > b_T^w - s^w(x_j)$. In this case, the intermediary wins the auction and firm i's ex-post net payoff is then

$$\begin{aligned}\tilde{u}_i(\bar{s}, \boldsymbol{\sigma}^w_{-i}, \boldsymbol{x}|(A,A)) = x_i - \frac{\bar{s}.b^w_T}{\bar{s}+s^w(x_j)} - t &< x_i - \frac{\left(b^w_T - s^w(x_j)\right).b^w_T}{b^w_T} - t \\ &= x_i - 2(1+\lambda) + \underline{k}x_j - t \\ &< x_i - (1+\lambda)(2-x_j) - t \\ &< -t = \tilde{u}_i(\boldsymbol{\sigma}^w, \boldsymbol{x}|(A,A))\end{aligned}$$

which ensures that firm i does not regret losing. Finally, since firms are symmetric, a similar reasoning holds for firm $j \neq i$.

Hence, in such equilibria, the intermediary's intervention fails to provide firms with safety from litigation brought by the troll. As compared to the patent auction without intermediation, firms are actually strictly worse off since they end up with a strictly negative payoff due to the fact that the membership fee is non-refundable. On the one hand, if the troll sticks to an aggressive bidding behavior, then the collective patent purchase through firms' contributions is not feasible as the troll's bid exceeds firms' *aggregate* value for the patent: $x_1 + x_2 < 2(1+\lambda) = b^w_T \ \forall \boldsymbol{x} \in [0,1]^2$. It follows that any vector of contributions $\boldsymbol{s}$ such that $b_I > b^w_T$ would make firms strictly worse off. Thus, firms optimally react by shading their contributions down so that the intermediary loses the auction for sure. On the other hand, the free-rider issue inherent to the intermediary's funding mechanism greatly impairs his bid, which in turn spurs the troll's aggressive bidding behavior.

One way to circumvent this severe free-rider problem is for firms to pledge aggressive contributions so that the troll always prefers to lose the auction from an ex-post perspective. Namely, if each firm contributes $s(x_i) > (1+\lambda)x_i$, then the troll finds it optimal to *always* lose the auction since $(1+\lambda)(x_1+x_2) - b_I < 0$. In particular, submitting a nill bid is a best response for the troll. Importantly, even though the intermediary's refund mechanism does not effectively alleviate the free-rider issue inherent to the contribution game, it is necessary for the existence of an equilibrium in which the intermediary outbids the troll with two contributors as it enables firms to play aggressively so as to drive the troll's bid down. The next result formalizes.

Proposition 11. *Suppose that both firms join the intermediary. There exists a continuum of ex-post equilibria in which the intermediary always wins the auction and where firms pledge aggressive contributions* $s^a(x_i) = \bar{k}x_i$ *with* $\bar{k} > 1+\lambda$, $i = 1,2$, *while the troll bids* $b^a_T = 0$.

By adopting an aggressive behavior, firms are fully protected from any risk of litigation for patent infringement by receiving a non-exclusive license to the patent acquired by the intermediary. While they have no means to win the auction and always face costly litigation when individually competing against the troll in any ex-post equilibrium of the unintermediated auction (see Proposition 6), the intermediary's intervention may effectively overturn this negative result by encouraging and aggregating aggressive contributions. Proposition 11 suggests that intermediaries are more successful at acquiring patents infringed by a larger subset of their members, which may occur for patents with a broader scope for instance.

Nevertheless, since the price is set by the second highest bid, it follows that the seller's revenue is strongly undermined whenever the intermediary wins the patent auction. In this case, the patentholder does not receive any monetary compensation for the infringement of his rights, thereby raising concerns about the competition between both offensive and defensive NPEs in acquiring patents. While the patent troll alone effectively deters efficient infringement, the intervention of an intermediary who aims to counter trolls' activity leads to zero revenue for the patentholder.

4.3 The intermediary's problem

Observe first that the intermediary finds it optimal to induce aggressive contributions whenever both firms accept his offer. Indeed, since the patent purchase is not feasible through the contribution of a sole firm (cf. Proposition 8), firms' perceived probability that the intermediary wins the auction would otherwise be zero, so that incurring the non-refundable membership fee would then be strictly unprofitable. Consequently, firms would turn his offer down for any threshold signal and strictly positive subscription fee, yielding zero profit to the intermediary.

Hence, the intermediary chooses a threshold signal $\hat{x} \in (0, 1)$ and a strictly positive membership fee t such that any firm holding a signal above the threshold finds it optimal to become a member. In this respect, a firm finds it profitable to accept the offer if her expected benefit from joining the intermediary, defined as her updated probability that the intermediary wins the auction times her value for the patent, net of the membership fee, exceeds her payoff upon rejecting the offer. Since the intermediary fails to acquire the patent with only one contributor, it follows that firm i's updated probability that the intermediary wins the auction, conditional on holding a signal $x_i \in [\hat{x}, 1]$, simply equals the probability that firm j's signal exceeds the threshold $\hat{x}$ as well, and that her payoff upon rejecting the offer is zero regardless of whether the other firm accepts. Thus, the participation constraint of (say) firm i writes

$$(1 - F(\hat{x}))x_i - t \geq 0 \quad \forall x_i \in [\hat{x}, 1]\,,\ i = 1, 2$$

The intermediary therefore chooses a threshold signal $\hat{x}$ and a membership fee t that maximize his ex-ante expected profit, which equals the total expected membership fees[26], subject to

[26]Recall that the intermediary's expected number of members is simply equal to

$$\mathbb{E}(\sum_{i=1,2} \mathbb{1}(X_i > \hat{x})) = \sum_{i=1,2} P(X_i > \hat{x}) = 2(1 - F(\hat{x}))$$

firms' participation constraint. That is, the intermediary's problem writes

$$\max_{(\hat{x},t)\in[0,1]\times\mathbb{R}_+} 2\,(1-F(\hat{x}))\,t$$
$$s.t. \quad (1-F(\hat{x}))\,x_i - t \geq 0 \quad \forall x_i \in [\hat{x},1]$$

Since optimality requires that the participation constraint binds for the threshold signal $\hat{x}$, the membership fee is given by $t = (1-F(\hat{x}))\hat{x}$. Plugging this into the intermediary's objective and rearranging yields $\Pi_I(\hat{x}) = 2\hat{x}(1-F(\hat{x}))^2$. One can easily check that the intermediary's profit function Π_I is strictly quasi-concave in $\hat{x}$, and that the unique optimal threshold signal is implicitely defined by

$$\frac{1-F(\hat{x}^*)}{\hat{x}^* f(\hat{x}^*)} = 2 \tag{1}$$

The intermediary finds it optimal to screen out low signal firms by proposing a strictly positive membership fee such that firms holding a signal above the threshold $\hat{x}^*$ find it profitable to become members. Put differently, the intermediary specializes in defensive patent acquisitions targeting firms facing higher risks of litigation for patent infringement.

Nevertheless, observe that firms are engaged in a coordination game with the troll whenever both of them join. Indeed, our previous analysis characterizes two classes of ex-post equilibria in the continuation game Γ_2 yielding two opposite outcomes (see subsection 4.2). While firms get a strictly positive ex-post payoff when they both pledge aggressive contributions and the troll bids zero, firms end up strictly worse off if the equilibrium profile in which they play conservatively while the troll bids aggressively instead prevails since the membership fee is non-refundable, so that it is optimal to accept (resp. reject) the intermediary's offer in the former (resp. latter) case. Therefore, if both firms hold a signal greater than the threshold $\hat{x}^*$, the whole game admits the two following classes of equilibria

$$\mathcal{E}_w = ((\hat{x}^*, t^*), (R,R), \boldsymbol{\sigma}^w) \text{ and } \mathcal{E}_a = ((\hat{x}^*, t^*), (A,A), \boldsymbol{\sigma}^a)$$

As these two classes of equilibria yield two opposite outcomes, we now ask whether either equilibrium constitutes an "unreasonable" prediction when both firms hold a signal above the threshold $\hat{x}^*$. Intuitively, if both firms decide to join the intermediary, they are giving up a certain payoff of zero. Since they get a strictly negative payoff when contributing cautiously, the troll should therefore expect firms to play aggressively, and bid zero himself. Invoked here is the idea of *forward induction*[27] (Kohlberg and Mertens, 1986) which says that the play leading to the continuation game Γ_2 conveys information about firms' intentions to play

[27] This equilibrium refinement is commonly used in coordination games exhibiting multiple equilibria. Experimental evidence in support of forward induction has been provided for a wide range of coordination games (such as the Battle of Sexes). See for instance Cooper et al. (1992), Van Huyck et al. (1993), and Shahriar (2014).

subsequently. Hence, upon observing $|\mathcal{I}| = 2$, the troll should assign probability zero to firms pledging cautious contributions in equilibrium. In other words, the equilibrium $\mathcal{E}_a$ is robust to forward induction. The next result summarizes these findings.

Proposition 12. *The equilibrium of the intermediated auction surviving forward induction entails the following:*

1. *The intermediary's optimal pair of threshold signal and membership fee $(\hat{x}^*, t^*)$ is unique and implicitely defined by*

$$\frac{1 - F(\hat{x}^*)}{\hat{x}^* f(\hat{x}^*)} = 2 \quad \textit{and} \quad t^* = (1 - F(\hat{x}^*))\hat{x}^* , \quad \textit{with } \hat{x}^* \in (0, 1).$$

and such that a firm accepts his offer if and only if she holds a signal in $[\hat{x}^*, 1]$, and rejects if and only if her signal instead lies in $[0, \hat{x}^*)$.

2. *If both firms accept, then the intermediary always outbids the troll, whereas if either one or both firms reject, then the troll always wins the auction.*

The intermediary's equilibrium ex-ante probability of winning the auction, given by $P_I^* = [1 - F(\hat{x}^*)]^2$, is thus strictly positive (and less than one) since $\hat{x}^* \in (0, 1)$. From an ex-ante perspective, the intermediary therefore partially hampers the troll's supremacy when it comes to acquiring threatening patents with the intent to engage in litigious activity against firms. The effectiveness of his intervention to protect firms against litigation brought by the troll is nonetheless mitigated by the fact that the collective patent purchase is feasible *only if* both firms contribute. This further suggests that the true value of the patent for sale is higher whenever the intermediary acquires it as compared to the troll. This comes from the fact that the intermediary wins only if both firms hold a signal above the threshold $\hat{x}^*$, whereas the troll wins otherwise. Figure 3 illustrates the outcome of the patent auction as well as the intermediary's number of members in the equilibrium surviving forward induction in the space of signal realizations (x_1, x_2). The red (resp. blue) area shows combinations of signal realizations such that the troll (resp. the intermediary) wins the patent auction in equilibrium.

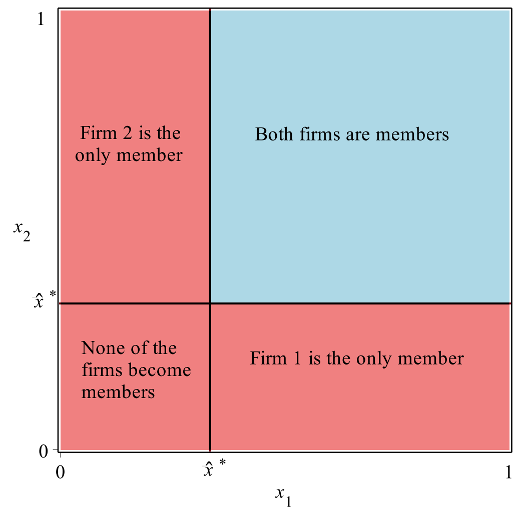
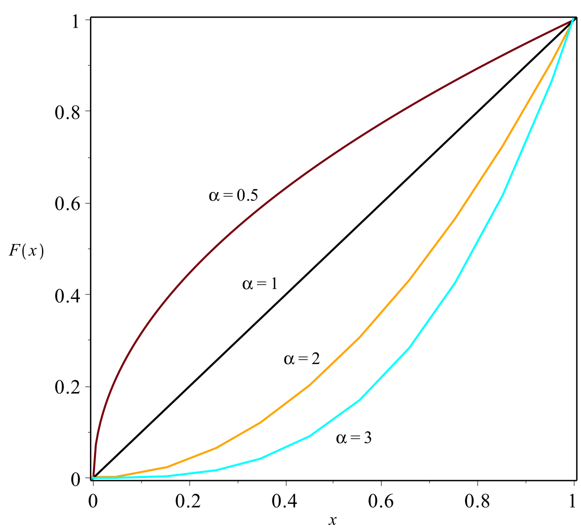


Figure 3: Intermediary's number of members and outcome of the patent auction in equilibrium

Example. (Continued) Suppose that signals are independently and identically to the power function distribution. As Figure 4 shows, higher values of the parameter α shift the distribution F to the right in the sense of first-order stochastic dominance. That is, from an ex-ante perspective, firms are more likely to be highly exposed to the patent for sale for higher values of α.



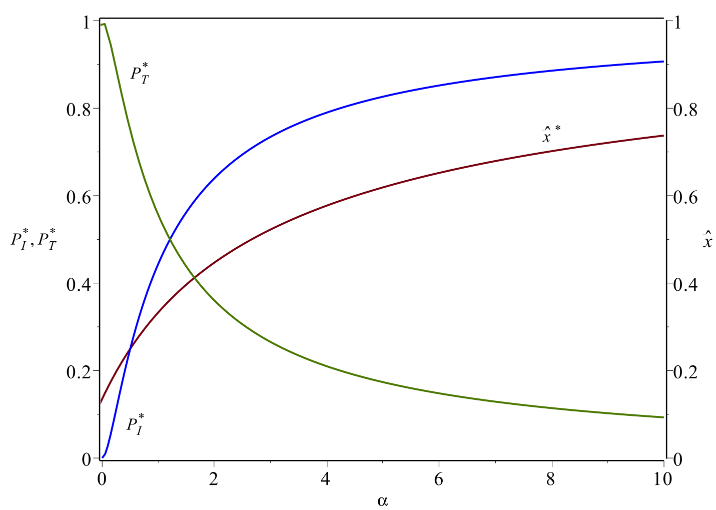


Figure 4: Left: Power function distribution for different values of the parameter α. Right: NPEs' ex ante equilibrium probabilities to win the patent and optimal threshold signal as functions of the parameter α.

The optimal threshold signal as defined by Eq. (1) is given by

$$\hat{x}^* = \left[\frac{1}{2\alpha+1}\right]^{1/\alpha}$$

and straightforward computations show that $\frac{d\hat{x}^*}{d\alpha} > 0$ for all $\alpha > 0$. Intuitively, since firms cannot acquire the patent for sale when individually competing against the troll in the auction, demand for the intermediary's services increases with their likelihood of facing a high degree of exposure to the patent for sale, which results in a higher expected number of members. In turn, the intermediary finds it optimal to screen firms out more severely, by charging a higher up-front membership fee. Finally, the intermediary's ex ante probability to win is $P_I = [1 - x^\alpha]^2$, while the troll's is $P_T = 1 - P_I = 1 - [1 - x^\alpha]^2$. Evaluating these at the equilibrium threshold signal yields

$$P_I^* = \frac{4\alpha^2}{(2\alpha+1)^2} \quad \text{and} \quad P_T^* = \frac{4\alpha+1}{(2\alpha+1)^2}$$

where $\frac{dP_I^*}{d\alpha} > 0$ and $\frac{dP_T^*}{d\alpha} < 0$, which is consistent with the idea that the intermediary is more likely to acquire patents that represent a higher threat of infringement to its members.

Albeit its positive effect on producers facing high risks of litigation in protecting them against trolls' activity, the intermediary strongly undermines the seller's revenue. Indeed, the presence of the intermediary harms the patentholder through two channels. First, the troll's response to firms' aggressiveness whenever both contribute yields zero revenue to the seller. Also, because he bids on behalf of his members, the intermediary's intervention softens competition to acquire the patent for sale by decreasing the number of bidders in the auction. The combination of these two negative effects therefore suggests that the competition between offensive and defensive non-practicing entities in contests for strategic patent acquisitions harms sellers of those patents that are likely to be infringed.

5 Concluding remarks

The aim of this paper is to study the emergence of non-practicing entities and their interaction in the market for patents, who acquire patents with no aim to engage in innovative activities. While patent trolls seek to monetize their acquired patents through the threat of litigation against alleged infringers, intermediaries instead intend to provide their affiliated firms with safety to operate from trolls' litigious activity by buying out patents that would otherwise fall in trolls' hands.

We develop a model of patent acquisition through an auction incorporating both patent trolls and intermediaries. We highlight trolls' greater ability to preempt patents that represent a threat upon enforcement for patent infringement as compared to producing firms due to their immunity to countersuits. We find that firms have no means to protect themselves against threatening patents when individually competing against the troll in the auction for patent buyout and that the seller's revenue may increase in response of the troll's participation in the auction. In particular, our results suggest that patent trolls effectively deter producing firms from willfully infringing patents (also known as "efficient infringement"), and that sellers of patents that have a mild likelihood of being upheld and found infringed by courts benefit from patent trolls. We then introduce an intermediary who, in exchange for an endogenous membership fee, participates in the auction on firms' behalf by aggregating their bids. Since the patent is collectively financed through voluntary individual contributions, firms tend to free ride on other contributors. While the intermediary's probability to outbid the troll in the auction is strictly positive, the collective action issue inherent to his funding mechanism greatly hampers his performance in the auction and undermines the seller's revenue. Our results further suggest that the competition between offensive and defensive NPEs in contests for patents harms the patentholder's revenue, thereby raising concerns on incentives to innovate in the first place.

The results of the two present models taken together provide a convincing theoretical foundation for understanding the crucial role played by both offensive and defensive non-practicing in the now frequent contests for patents.

Appendix

Claim. If $\frac{d}{dx}\left(\frac{1-F(x)}{f(x)}\right) \leq 0$, then $\frac{d}{dx}\left(\frac{1-F(x)}{xf(x)}\right) < 0$ for all $x \in [0,1]$.

Proof. We have that

$$\frac{d}{dx}\left(\frac{1-F(x)}{f(x)}\right) \leq 0 \Leftrightarrow -\left[f(x)\right]^2 \leq \left[1-F(x)\right] f'(x)$$

Multiplying both sides by $x \in [0,1]$ yields

$$-x\left[f(x)\right]^2 \leq \left[1-F(x)\right] x f'(x)$$

Since the density function f is strictly positive everywhere by assumption, it follows that

$$-x\left[f(x)\right]^2 < \left[1-F(x)\right]\left(xf'(x)+f(x)\right)$$

or, equivalently,

$$\frac{d}{dx}\left(\frac{1-F(x)}{xf(x)}\right) < 0$$

□

Proof of Proposition 1.

Let β_e denote firms' common equilibrium strategy, which is assumed to be continuous and strictly increasing in the signal they receive, and $\phi(b) \equiv \beta_e^{-1}(b)$ firms' equilibrium inverse bidding function, where $\phi : [\beta_e(0), \beta_e(1)] \to [0, 1]$ is continuous and strictly increasing. Throughout, we shall say that the troll bids aggressively whenever he submits any $b > \beta_e(1)$.

Furthermore, we reorder X_1, X_2 and let Y_1, Y_2 denote the rearranged signals so that $Y_1 \geq Y_2$, where Y_k is distributed according to F_k, $k = 1, 2$, given by[28]

$$F_1(y_1) = [F(y_1)]^2 \quad \text{and} \quad F_2(y_2) = 2F(y_2) - [F(y_2)]^2$$

with associated marginal densities $f_1(y_1) = 2f(y_1)F(y_1)$ and $f_2(y_2) = 2f(y_2)(1 - F(y_2))$, and joint density $f_{1,2}(y_1, y_2) = 2f(y_1)f(y_2)$ if $0 \leq y_2 \leq y_1 \leq 1$ and 0 elsewhere.

(i) We first show that firms' equilibrium bidding strategies are bounded above by $\bar{w}(x_i) = 2x_i$ for all $x_i \in [0, 1]$, $i = 1, 2$. Consider (say) firm i and let us derive her maximum willingness-to-pay for the patent, $w_i(x_i)$, defined as the tying bid at which firm i is indifferent between winning and losing. Two cases need to be considered. If the tying bidder is the troll, then firm i infers that $X_j < x_i$ so that her maximum willingness-to-pay is given by

$$\mathbb{E}[v(X_i, X_j)|X_i = x_i, X_j < x_i] - w_i(x_i) = 0 \quad \Leftrightarrow w_i(x_i) = x_i + \mathbb{E}[X_j|X_j < x_i] \equiv \underline{w}(x_i)$$

Instead, if the tying bidder is firm j, then firm i infers that $X_j = x_i$ by symmetry of bidding strategies. Therefore, her maximum willingness-to-pay is given by

$$\mathbb{E}[v(X_i, X_j)|X_i = x_i, X_j = x_i] - w_i(x_i) = 0 \quad \Leftrightarrow w_i(x_i) = 2x_i \equiv \bar{w}(x_i)$$

Thus, firms' equilibrium bids are indeed bounded above by $\bar{w}(x_i) = 2x_i$ $\forall x_i \in [0, 1]$, $i = 1, 2$.

Next, for given firms' equilibrium strategies $\beta_e(x_i)$, let us define $G(\lambda, b)$ as the troll's expected payoff upon winning when bidding any $b > 0$. Since the troll wins if $b > \beta_e(Y_1) \Leftrightarrow Y_1 < \phi(b)$, we have

$$G(\lambda, b) \triangleq \mathbb{E}\left[((1 + \lambda)(Y_1 + Y_2) - \beta_e(Y_1))\,\mathbb{1}_{Y_1 \leq \phi(b)}\right] \tag{2}$$

Consider first the case where firms' equilibrium strategy is such that $x_i \leq \beta_e(x_i) \leq \underline{w}(x_i)$ for any $x_i \in [0, 1]$, $i = 1, 2$, where the first inequality comes from the fact that we focus on

[28]See for instance Krishna (2009), pp. 281-284.

equilibria in undominated strategies. Note that

$$\mathbb{E}(\underline{w}(Y_1)) = \int_0^1 \left(y_1 + \int_0^{y_1} t \frac{f(t)}{F(y_1)} \mathrm{d}t \right) f_1(y_1)\mathrm{d}y_1 = \int_0^1 y_1 f_1(y_1)\mathrm{d}y_1 + 2\int_0^1 t\,(1-F(t))\,f(t)\mathrm{d}t$$

$$= \int_0^1 y_1 f_1(y_1)\mathrm{d}y_1 + \int_0^1 y_2 f_2(y_2)\mathrm{d}y_2 = \mathbb{E}(Y_1+Y_2)$$

Straightforwardly, we have that $\mathbb{E}\left[\underline{w}(Y_1)\mathbb{1}_{Y_1\le\phi(b)}\right] = \mathbb{E}\left[(Y_1+Y_2)\mathbb{1}_{Y_1\le\phi(b)}\right]$. The troll's expected payoff upon winning when bidding $b > 0$ is therefore given by

$$\begin{aligned} G(\lambda,b) &\geq \mathbb{E}\left[((1+\lambda)(Y_1+Y_2) - \underline{w}(Y_1))\,\mathbb{1}_{Y_1\le\phi(b)}\right] \\ &= \lambda\mathbb{E}\left[(Y_1+Y_2)\mathbb{1}_{Y_1\le\phi(b)}\right] \\ &\geq 0 \quad \forall\lambda\in[0,1] \end{aligned}$$

Hence, the troll always prefers to win for any $\lambda \in [0,1]$ whenever firms' equilibrium strategy lies in $[x_i, \underline{w}(x_i)]$ for all $x_i \in [0,1]$, $i = 1,2$, so that bidding aggressively is optimal as it ensures winning with probability one. In particular, $b = 2(1+\lambda)$ is a best response.

Consider now the case where $\underline{w}(x_i) < \beta_e(x_i) \leq \bar{w}(x_i)$. First, we show that winning is no longer profitable for the troll whenever his private-value advantage λ falls below a cutoff $\hat{\lambda} \in (0,1)$ as defined hereafter. From Eq. (2), observe that

$$G(0,b) = \mathbb{E}\left[(Y_1+Y_2-\beta_e(Y_1))\,\mathbb{1}_{Y_1\le\phi(b)}\right] < \mathbb{E}\left[(Y_1+Y_2-\underline{w}(Y_1))\,\mathbb{1}_{Y_1\le\phi(b)}\right] = 0$$

and

$$\begin{aligned} G(1,b) &= \mathbb{E}\left[(2(Y_1+Y_2) - \beta_e(Y_1))\,\mathbb{1}_{Y_1\le\phi(b)}\right] \\ &\geq \mathbb{E}\left[(2(Y_1+Y_2) - \bar{w}(Y_1))\,\mathbb{1}_{Y_1\le\phi(b)}\right] \\ &= 2\mathbb{E}\left[Y_2\mathbb{1}_{Y_1\le\phi(b)}\right] \end{aligned}$$

which is strictly positive provided that $b > 0$. Moreover, we have that $\frac{\partial G}{\partial\lambda} = \mathbb{E}\left[(Y_1+Y_2)\,\mathbb{1}_{Y_1\le\phi(b)}\right] > 0$ for any $\phi(b) > 0$. Holding b fixed, since G is continuous in λ, there exists a unique $\hat{\lambda} \in (0,1)$ such that $G(\hat{\lambda}, b) = 0$. Therefore, it follows that $G(\lambda,b) \geq 0$ for any $\lambda \in [\hat{\lambda}, 1]$ so that bidding aggressively is optimal for the troll since winning is always profitable whenever his private-value advantage exceeds the cutoff $\hat{\lambda}$. In particular, submitting $b = 2(1+\lambda)$ constitutes a best response. Instead, since $G(\lambda,b) < 0$ for all $\lambda \in [0,\hat{\lambda})$, the troll strictly prefers to lose the auction which is ensured by bidding zero.

(ii) We first show that if the troll pledges zero in equilibrium, then firms' best response is to bid twice their signal. To see this, consider, say, firm i and suppose that firm $j \neq i$ pledges $\beta_e(x_j) = 2x_j$. By playing $\beta_e(x_i) = 2x_i$, firm i wins if $x_i > x_j$. She gets $x_i + x_j - 2x_j =$

$x_i - x_j > 0$ which ensures that she does not regret winning, and given the auction format, submitting a higher bid does not improve her payoff. Instead, if $x_i < x_j$, then she loses and does not suffer from ex-post regret either as $x_i + x_j - 2x_j = x_i - x_j < 0$. Hence, playing $\beta_e(x_i) = 2x_i$ for all $x_i \in [0,1]$, $i = 1, 2$ is optimal and satisfies the no ex-post regret property. Finally, suppose that the troll instead bids $b = 2(1+\lambda)$ in equilibrium so that he always wins. Firms do not regret losing since $x_i + x_j \leq 2 \leq b$, which completes the proof of the second part. □

Proof of Proposition 2.

Let us first consider the troll and suppose that firms play according to $b(.)$. By bidding b_T, the troll always wins and gets

$$\mathbb{E}\left[(1+\lambda)(Y_1+Y_2) - b(Y_1)\right] \geq \mathbb{E}\left[(1+\lambda)(Y_1+Y_2 - \underline{w}(Y_1))\right] = 0$$

from the proof of Proposition 1. Given the auction format, submitting a higher bid does not improve his payoff, while lowering his bid triggers a positive probability to lose the auction. Hence, bidding aggressively ensures that the troll outbids firms and in particular, $b_T = 2(1+\lambda)$, constitutes a best response.

We now turn to (say) firm i and show that bidding $b(x_i)$ is a best response and satisfies the no ex-post regret property. Suppose that the troll and firm $j \neq i$ play according to the aforementionned strategies. By pledging $b(x_i)$, firm i always loses the auction since $b(x_i) < b_T$, and gets zero payoff. She does not suffer from ex-post regret since winning would instead yield $x_1 + x_2 - b_T = x_1 + x_2 - 2(1+\lambda) \leq 0 \;\; \forall \lambda \in [0,1],\; \forall \boldsymbol{x} \in [0,1]^2$. Obviously, either lowering her bid or bidding any $a_i \in (b(x_i), b_T)$ does not change the outcome or her payoff. Since firms' strategies are symmetric, a similar argument holds for firm j, $j \neq i$. Finally, note that these equilibrium strategies are undominated since $b_T = 2(1+\lambda) = V_T(\mathbf{1})$ and $V_i(x_i, 0) = x_i \leq b(x_i) < x_i + 1 = V_i(x_i, 1)$ for all $x_i \in [0,1]$, $i = 1, 2$. □

Proof of Proposition 3.

Suppose that firms play according to the proposed bidding strategies. If $\lambda < \hat{\lambda}$, bidding $b_T = 0$ ensures that the troll always loses and gets zero payoff. Bidding aggressively, that is any $b'_T \geq b(1) = 2$ would instead yield $\mathbb{E}\left[(1+\lambda)(Y_1+Y_2) - 2Y_1\right] < 0 \Leftrightarrow \lambda < \frac{\mathbb{E}(Y_1-Y_2)}{\mathbb{E}(Y_1+Y_2)} \equiv \hat{\lambda}$. Hence, bidding zero is indeed a best response for the troll. Instead, if $\lambda \geq \hat{\lambda}$, the troll always wins when bidding $b_T = 2(1+\lambda)$. His payoff is then

$$\mathbb{E}\left[(1+\lambda)(Y_1+Y_2) - b(Y_1)\right] \geq \mathbb{E}\left[(1+\lambda)(Y_1+Y_2) - 2Y_1\right] \geq 0 \Leftrightarrow \lambda \geq \frac{\mathbb{E}(Y_1-Y_2)}{\mathbb{E}(Y_1+Y_2)} \equiv \hat{\lambda}$$

which ensures that bidding aggressively is indeed a best response. We now turn to, say, firm i and suppose that the troll and firm $j \neq i$ play the proposed strategies. If $\lambda < \hat{\lambda}$, then by bidding $b(x_i) = 2x_i$, firm i wins against firm j if $x_i > x_j$ and gets $x_i + x_j - 2x_j = x_i - x_j > 0$, which ensures that she does not regret winning, and given the auction format, increasing her bid does not improve her ex-post payoff. If $\lambda \geq \hat{\lambda}$, then firm i always loses when bidding $b(x_i)$ and gets a zero ex-post payoff. She does not regret losing as $x_i + x_j \leq 2 < 2(1+\lambda) = b_T$. By symmetry, a similar argument holds for firm j. Finally, since $b_T \in \mathcal{A}_T$ and $b(x_i) \in \mathcal{A}_i(x_i)$ for all $i = 1, 2$, the equilibrium does not involve the use of weakly dominated strategies. □

Proof of Lemma 2.

Suppose that $F \underset{r.h.}{\succ} \mathcal{U}$. We have that $\bar{\beta}(x) \geq x$ for all x if and only if

$$\int_0^x t\frac{f(t)}{F(x)}\mathrm{d}t \geq \frac{x}{2} = \int_0^x \frac{t}{x}\mathrm{d}t$$

Using integration by parts, this is equivalent to

$$\int_0^x \frac{t}{x}\mathrm{d}t \geq \int_0^x \frac{F(t)}{F(x)}\mathrm{d}t$$

A sufficient condition for this inequality to hold is $\frac{F(t)}{t} \leq \frac{F(x)}{x}$ for all $t \leq x$. This is equivalent to, for all t,

$$\frac{d}{dt}\left(\frac{F(t)}{t}\right) \geq 0 \Leftrightarrow \frac{f(t)}{F(t)} \geq \frac{1}{t} \Leftrightarrow F \underset{r.h.}{\succ} \mathcal{U}$$

which holds by assumption. □

Proof of Proposition 4.

1. It directly follows from the fact that the seller gets zero payoff under the EI scenario (from subsection 3.1).

2. Let ΔR denote the difference between the seller's expected equilibrium revenue when the troll participates in the auction and his expected equilibrium revenue when firms are the only bidders under the GFB scenario. In the absence of the troll, firms' unique symmetric equilibrium strategy is $\beta^s(x_i) = 2x_i$ for all x_i, $i = 1, 2$, so that the seller's expected equilibrium revenue is equal to $2\mathbb{E}(Y_2)$. We have that

$$\begin{aligned}\mathbb{E}(\bar{\beta}(Y_1)) = 2\int_0^1 \left(\int_0^{y_1} t\frac{f(t)}{F(y_1)}\mathrm{d}t\right) f_1(y_1)\mathrm{d}y_1 &= 4\int_0^1 \left(\int_0^{y_1} tf(t)\mathrm{d}t\right) f(y_1)\mathrm{d}y_1 \\ &= 4\int_0^1 tf(t)\left(1 - F(t)\right)\mathrm{d}t = 2\mathbb{E}(Y_2)\end{aligned}$$

Therefore, we have that $\Delta R \geq 0$ whenever firms' equilibrium bids lie above $\bar{\beta}(x_i)$. Next, we must ensure that $\bar{\beta}(x_i)$ belongs to firms' set of symmetric undominated equilibrium bids $[x_i, 2x_i]$. Straightforwardly, we have that $\bar{\beta}(x_i) \leq 2x_i$ for all $x_i \geq 0$. Instead, we have that:

(i) From Lemma 2, $\bar{\beta}(x_i) \geq x_i$ for all x_i when $F \underset{r.h.}{\succ} \mathcal{U}$. Hence, it directly follows that $\Delta R < 0$ if $b(x_i) \in [x_i, \bar{\beta}(x_i))$ and $\Delta R \geq 0$ if $b(x_i) \in [\bar{\beta}(x_i), 2x_i]$.

(ii) Similarly to the proof of Lemma 2, it can be shown that $\bar{\beta}(x_i) \leq x_i$ for all x_i if $\mathcal{U} \underset{r.h.}{\succ} F$. Therefore, $\Delta R \geq 0$ for any $b(x_i) \in [x_i, 2x_i]$. □

Proof of Lemma 3.

Suppose first that $\lambda \geq \hat{\lambda}$ and $\hat{\lambda} \leq \Delta(\boldsymbol{x})$. Then the troll always outbids firms and his ex-post payoff upon winning is $(1+\lambda)(x_1+x_2) - 2x_1 \leq 0 \Leftrightarrow \lambda \leq \frac{x_1-x_2}{x_1+x_2} = \Delta(\boldsymbol{x})$. Thus, the troll suffers from ex-post regret upon winning whenever $\lambda \in [\hat{\lambda}, \Delta(\boldsymbol{x})]$. Similarly, suppose that $\lambda < \hat{\lambda}$ and $\hat{\lambda} \geq \Delta(\boldsymbol{x})$ so that the troll loses the auction. His ex-post payoff is then $(1+\lambda)(x_1+x_2) - 2x_1 \geq 0 \Leftrightarrow \lambda \geq \frac{x_1-x_2}{x_1+x_2} = \Delta(\boldsymbol{x})$, so that he suffers from ex-post regret upon losing for any $\lambda \in [\Delta(\boldsymbol{x}), \hat{\lambda})$. □

Proof of Proposition 5.

Let $\hat{\lambda}_F \equiv \frac{\mathbb{E}_F(Y_1-Y_2)}{\mathbb{E}_F(Y_1+Y_2)}$ and $\hat{\lambda}_G \equiv \frac{\mathbb{E}_G(Y_1-Y_2)}{\mathbb{E}_G(Y_1+Y_2)}$. From Lemma 3, the set of values of λ for which the troll suffers from ex-post regret upon winning (resp. losing) shrinks under the distribution F (resp. G) if $\hat{\lambda}_F \geq \hat{\lambda}_G$. Note that $\mathbb{E}(Y_1+Y_2) = \mathbb{E}(X_1+X_2)$ and $\mathbb{E}(Y_2) = 2\mathbb{E}(X_i) - \mathbb{E}(Y_1)$. Since G is a mean-preserving spread (m.p.s) of F, it follows that

$$\hat{\lambda}_F \geq \hat{\lambda}_G \Leftrightarrow \frac{\mathbb{E}_F(Y_1-Y_2)}{\mathbb{E}_F(Y_1+Y_2)} \geq \frac{\mathbb{E}_G(Y_1-Y_2)}{\mathbb{E}_G(Y_1+Y_2)} \Leftrightarrow \mathbb{E}_F(Y_1) \geq \mathbb{E}_G(Y_1)$$

Thus, it suffices to show that the distributions F_1 and G_1 of the first-order statistics inherit the stochastic order of the original distributions F and G. By definition, since G is a m.p.s. of F, F second-order stochastically dominates G ($F \succ_2 G$), i.e.,

$$\int_0^t [G(t) - F(t)]\,\mathrm{d}t \geq 0 \;\Rightarrow \int_0^t [G(t) - F(t)]\,.\,[G(t) + F(t)]\,\mathrm{d}t \geq 0 \quad \forall t \in [0,1]$$

which is equivalent to

$$\int_0^t \left[G(t)^2 - F(t)^2\right]\mathrm{d}t = \int_0^t [G_1(t) - F_1(t)]\,\mathrm{d}t \geq 0 \Leftrightarrow F_1 \succ_2 G_1$$

Hence, $\mathbb{E}_F(Y_1) \geq \mathbb{E}_G(Y_1) \Leftrightarrow \hat{\lambda}_F \geq \hat{\lambda}_G$. □

Proof of Proposition 6.

Assume not. Since from Proposition 1, the troll bids either zero or $b_T = 2(1+\lambda)$ in any equilibrium in which firms employ symmetric strategies, it is sufficient to restrict attention to the case where the troll's equilibrium bid is $b_T = 0$. From the proof of Proposition 3, firms' best-response is then to pledge $b(x_i) = 2x_i$, which ensures that they do not suffer from ex-post regret regardless of the outcome of the auction. Thus, for $(b_T, b(x_1), b(x_2))$ to form an ex-post equilibrium, it must be that the troll does not suffer from ex-post regret upon losing. That is, the following inequality must hold for any pair $(x_1, x_2) \in [0,1]^2$

$$0 \geq (1+\lambda)(x_1 + x_2) - 2x_1 \tag{3}$$

But notice that for $x_2 \to x_1$, then the right-hand-side of Eq. (3) goes to $2(1+\lambda)x_1 - 2x_1 > 0$ for any $\lambda > 0$, a contradiction. □

Proof of Proposition 7.

($\Rightarrow$) Suppose that the strategies b_T and $b(x_i)$ constitute an ex-post equilibrium. Then, it must be that

$$u_T(b_T, b(x_1), b(x_2), \boldsymbol{x}) = (1+\lambda)(x_1 + x_2) - \gamma.\max\{x_1, x_2\} \geq u_T(a_T, b(x_1), b(x_2), \boldsymbol{x})$$

for all $\boldsymbol{x} \in [0,1]^2$ and all $a_T \in \mathcal{A}_T = [0, 2(1+\lambda)]$. Suppose that the troll instead bids any $a_T < 2(1+\lambda)$. Given the auction format, the troll's payoff only changes if $a_T < \gamma.\max\{x_1, x_2\}$, in which case the troll loses the auction and gets zero payoff. Suppose w.l.o.g. that $x_1 \geq x_2$, we have that:

$$\begin{aligned}(1+\lambda)(x_1+x_2) - \gamma.\max\{x_1,x_2\} = (1+\lambda)(x_1+x_2) - \gamma x_1 \geq 0 \quad &, \forall \boldsymbol{x} \in [0,1]^2 \\ \Rightarrow\ (1+\lambda)x_1 \geq \gamma x_1 \quad &, x_1 \in [0,1] \\ \Leftrightarrow\ \lambda \geq \gamma - 1 \equiv \underline{\lambda} \quad & \end{aligned}$$

($\Leftarrow$) Suppose that $\lambda \geq \gamma - 1$. We now establish that the proposed strategies constitute an ex-post equilibrium. By bidding $b_T = 2(1+\lambda)$, the troll always wins and gets $(1+\lambda)(x_1 + x_2) - \gamma.\max\{x_1, x_2\} \geq \gamma(x_1+x_2) - \gamma.\max\{x_1,x_2\} \geq 0$ for any $\boldsymbol{x} \in [0,1]^2$, which ensures that he does not regret winning. Since the price he pays upon winning is the second highest bid, increasing his bid does not improve his payoff, while a lower bid triggers losing the auction resulting in a zero ex-post payoff. Likewise, firms do not regret losing since outbidding the troll would lead to $x_1 + x_2 - b_T \leq x_1 + x_2 - 2 \leq 0 \quad \forall \boldsymbol{x} \in [0,1]^2$. Finally, note that $b_T = 2(1+\lambda) = V_T(\mathbf{1})$ and $V_i(x_i, 0) = x_i \leq b(x_i) \leq x_i + 1 = V_i(x_i, 1)$ for all $x_i \in [0,1]$, $i = 1, 2$, which ensures that the equilibrium strategies are undominated. □

Proof of Proposition 8.

We first establish that contributing $s(x_1) = x_1$ is a best response for the firm. Observe that she always loses against the troll as $s(x_1) = x_1 \leq 1 < b_T$ and therefore gets a strictly negative ex-post payoff $-t$. She does not regret losing since outbidding the troll by pledging some $s_1' \geq b_T$ would instead yield $x_1 - s_1' - t \leq x_1 - b_T - t < -t$. Pledging instead any $s_1'' < x_1$ does not alter the outcome of the auction or her payoff. Hence, $s(x_1) = x_1$ is indeed a best response. Next, we show that bidding any $b_T \in (1, (1+\lambda)(1+\hat{x})]$ is a best response for the troll. By playing b_T, the troll always wins against the firm as $s(x_1) = x_1 \leq 1 < b_T$, and does not regret winning since he gets $(1+\lambda)(x_1+x_2) - x_1 \geq 0$ for any $\lambda \in [0,1]$, $x_2 \geq 0$. Given the auction format, submitting a higher bid does not improve is payoff upon winning. Thus, bidding any $b_T \in (1, (1+\lambda)(1+\hat{x})]$ constitutes a best response for the troll. □

Proof of Proposition 9.

Suppose that the profile of strategies $(s(x_1), s(x_2), b_T)$ constitutes an ex-post equilibrium of the continuation game Γ_2 for which the intermediary wins the auction, that is $s(x_1) + s(x_2) \geq b_T$. By contradiction, assume that excessive contributions are not refunded. Then, in equilibrium, it must be that firms pledge at most their signal, i.e. $s(x_i) \leq x_i$, as any higher contribution $s_i' > x_i$ yields $x_i - s_i' - t < -t$, making $s_i = 0$ a strictly profitable deviation. By definition, one must have that the troll does not regret losing the auction, that is, the following must hold for any pair $(x_1, x_2) \in [0,1]^2$

$$0 \geq (1+\lambda)(x_1+x_2) - (s(x_1) + s(x_2))$$

which contradicts $s(x_i) \leq x_i$ for all i. Hence, the troll strictly prefers to win the auction so that pledging any $b_T' > 2 \geq x_1 + x_2$ is a profitable deviation, a contradiction. □

Proof of Proposition 10.

Towards contradiction, assume first that there exists a profile of strategies $(s^e(x_1), s^e(x_2), b_T^e)$ that constitutes an ex-post equilibrium with $s^e(x_1) + s^e(x_2) > b_T^e > 0$. Consider, say, firm 1. By slightly lowering her contribution to $s_1' = s^e(x_1) - \epsilon$, with $\epsilon > 0$ such that $s_1' + s^e(x_2) \geq b_T^e$, she is strictly better off as

$$x_1 - \frac{s_1'.b_T^e}{s_1' + s^e(x_2)} - t > x_1 - \frac{s^e(x_1).b_T^e}{s^e(x_1) + s^e(x_2)} - t$$

$$\Leftrightarrow s^e(x_1)(s_1' + s^e(x_2)) > s_1'(s^e(x_1) + s^e(x_2))$$

$$\Leftrightarrow s^e(x_1) > s_1' = s^e(x_1) - \epsilon$$

Clearly, she finds it profitable to do so until the sum of contributions meets the troll's equilibrium bid, i.e. up to the point where $\underline{s}_1 + s^e(x_2) = b_T^e$. Tying with the troll then leads to

$$x_1 - \frac{\underline{s}_1 b_T^e}{\underline{s}_1 + s^e(x_2)} - t = x_1 - \frac{\underline{s}_1 b_T^e}{b_T^e} - t = x_1 - \underline{s}_1 - t$$

Two cases need to be considered: (a) if $x_1 - t \geq \underline{s}_1$, then firm 1 is strictly better off by deviating to $\underline{s}_1$, contradicting an equilibrium in symmetric strategies; (b) if $x_1 - t < \underline{s}_1$, then firm 1 strictly prefers to further reduce her contribution so that the troll wins the auction, a contradiction.

Consider now the case where $(s^e(x_1), s^e(x_2), b_T^e)$ forms an ex-post equilibrium with $s^e(x_1) + s^e(x_2) = b_T^e > 0$. By definition, it must be that all players are ex-post indifferent between winning and losing, i.e. $(s^e(x_1), s^e(x_2))$ must satisfy

$$\begin{cases} (1+\lambda)(x_1 + x_2) = s^e(x_1) + s^e(x_2) \\ x_i - t = s^e(x_i) \end{cases} \qquad \forall \boldsymbol{x} \in [0,1]^2,\ i = 1,2$$

Clearly, these two equalities are mutually exclusive. Hence, there is no pair $(s^e(x_1), s^e(x_2))$ such that tying with the troll at a positive price constitutes an ex-post equilibrium. This last argument completes the proof. □

Proof of Proposition 11.

Suppose that firms play according to s^a. By bidding $b_T^a = 0$, the troll always loses and gets

$$0 > (1+\lambda)(x_1 + x_2) - (s^a(x_1) + s^a(x_2))$$

which ensures that he does not regret losing. Consider now firm i, and assume that the troll and firm $j \neq i$ play the aforementioned strategies. By pledging $s^a(x_i)$, the intermediary always wins and firm i gets $x_i - t \geq -t$ so that she does not suffer from ex-post regret. Lowering her contribution to some s_i' only changes the outcome if both $s_i' = 0$ and $x_j = 0$ in which case she gets $-t < x_i - t$. Hence, pledging $s^a(x_i)$ indeed constitutes a best-response for firm i, $i = 1, 2$. □

Proof of Proposition 12.

Recall that the intermediary's objective writes $\Pi_I(\hat{x}) = 2\hat{x}(1 - F(\hat{x}))^2$. We first show that $\Pi_I(\hat{x})$ is strictly quasi-convave in $\hat{x}$. To this end, it is sufficient to show that the second derivative of $\Pi_I(\hat{x})$ is strictly negative whenever the first derivative equals zero. The FOC writes

$$\Pi_I'(\hat{x}) = 2(1 - F(\hat{x}))\,[1 - F(\hat{x}) - 2\hat{x} f(\hat{x})] = 0$$

Observe that the first derivative vanishes at both $\hat{x} = 1$ and $\hat{x}$ implicitely defined by $\frac{1-F(\hat{x})}{\hat{x}f(\hat{x})} = 2$. However, $\hat{x} = 1$ is not optimal since $\Pi_I(1) = 0$. The second derivative is

$$\Pi_I''(\hat{x}) = -4\left(f(\hat{x})\left[1 - F(\hat{x}) - \hat{x}f(\hat{x})\right] + (1 - F(\hat{x}))\left[\hat{x}f'(\hat{x}) + f(\hat{x})\right]\right)$$

Evaluating at $\frac{1-F(\hat{x})}{\hat{x}f(\hat{x})} = 2$ yields

$$\Pi_I''(\hat{x})\Big|_{FOC} = -4\left[\hat{x}(f(\hat{x}))^2 + (1 - F(\hat{x}))\left(\hat{x}f'(\hat{x}) + f(\hat{x})\right)\right] < 0$$

since the term into brackets in strictly positive under Assumption 1. Thus, $\Pi_I(\hat{x})$ is strictly quasi-concave in $\hat{x}$ and admits a unique interior argmax implicitely defined by

$$\frac{1 - F(\hat{x}^*)}{\hat{x}^* f(\hat{x}^*)} = 2.$$

Since firms' participation constraint binds at $x_i = \hat{x}$ for all $i = 1, 2$, we have that $t^* = (1 - F(\hat{x}^*))\hat{x}^*$. Finally, the proof of the following equivalences

$$\begin{cases} a_i^* = A & \Leftrightarrow x_i \in [\hat{x}^*, 1] \\ a_i^* = R & \Leftrightarrow x_i \in [0, \hat{x}^*) \end{cases} \quad \text{for all } i = 1, 2$$

directly follows from Definition 5 and the forward induction criterion, while the proof of the second part directly follows from the proofs of Propositions 6, 8 and 11. □

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