Received: 29 August 2017 | Accepted: 8 October 2017
DOI: 10.1111/jpet.12280

WILEY JOURNAL OF PUBLIC ECONOMIC THEORY

ARTICLE

Equal treatment and socially optimal R&D in duopoly with one-way spillovers

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We are very grateful to Rabah Amir for numerous conversations on the topic of this paper

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This paper examines the standard symmetric two-period R&D model with a deterministic one-way spillover structure: know-how flows only from the high R&D firm to the low R&D firm (but not vice versa). Though firms are *ex ante* identical, one obtains a unique asymmetric equilibrium (pair) in R&D investments, leading to interfirm heterogeneity in the industry. R&D cooperation by means of a joint lab is considered and compared to the non cooperative solution. The main part of the paper provides a second-best welfare analysis in which we show that the joint lab yields a socially optimal R&D level subject to an equal treatment (of firms) constraint, which also coincides with the noncooperative solution in the absence of spillovers. We also investigate the welfare costs of this equal treatment constraint and find that they can be quite significant.

1 | INTRODUCTION

In the context of nontournament models of research and development (henceforth, R&D) in which firms engage in cost-reducing innovation and then compete *à la* Cournot in the product market, it is widely recognized that exogenous knowledge spillovers create distortions in R&D investment decisions (see, e.g., the pioneering study by Spence, 1984). Along with other distortions associated with such models, such as imperfect competition, these spillovers cause a well-known significant wedge between the private and the public returns to R&D, leading to insufficient levels of R&D being supplied from the perspective of social optimality (see, e.g., Bernstein & Nadiri, 1988).

The bulk of the extant literature on imperfectly appropriable R&D focuses on deterministic multidirectional spillovers.[1] A fixed proportion (given by the spillover parameter) of every firm's R&D effort or benefit flows freely to the rivals. As argued by Kamien, Muller, and Zang(1992), such a spillover process is appropriate if the associated R&D process in the extant literature is implicitly assumed to be a multidimensional heuristic rather than a one-dimensional algorithmic process. Thus, it necessarily involves trial and error on the part of the firms, which follow potentially different sets of research paths and or approaches.

An exception to the deterministic multidirectional spillovers is proposed in the two studies by Amir (2000) and Amir and Wooders (2000), henceforth AW. These authors consider instead a stochastic directed spillover process whereby

[1] A small selection of papers includes Ruff (1969), Katz (1986), d'Aspremont and Jacquemin (1988, 1990), Kamien, Muller, and Zang (1992), Amir (2000), and Amir, Evstigneev, and Wooders (2003), among many others.

know-how may flow only from the more R&D-intensive firm to its rival.[2] In their model, spillovers are stochastic and admit only extreme realizations—either full or no spillovers occur with a given probability.[3] The latter probability is itself defined as the spillover parameter. As argued by AW, the idea underlying the assumption of a unidirectional spillover process is that it is a better approximation for the potential leakages that occur when the R&D process is either one-dimensional, that is, there is a single research path to achieve unit cost reductions, or multidimensional, in which case this spillover structure suggests that there is a well defined natural path to follow. In this context, the spillover parameter may be interpreted as being related to the length of patent protection, but also to a measure of the imitation lag.

The purpose of the present paper is to consider a deterministic one-way spillover process, which constitutes the certainty-equivalent version of AW's model. In other words, a fraction of the R&D lead of the leader (i.e., of the R&D differential between the two firms) flows to its rival with certainty. That fraction is itself defined as the spillover parameter, and ranges from zero (when R&D is a pure private good) to one (when R&D is a pure public good).

As to the rest of the model, we consider the standard two-period model of process R&D and product market competition with the said deterministic one-way spillover process. We adopt the common specification of linear market demand and identical linear firms' cost functions to facilitate closed-form solutions and comparability of outcomes with past literature.

We now give an overview of the main results of the paper and some general discussion. Though firms are *ex ante* identical, one obtains a unique pair of asymmetric equilibria so that the roles of R&D innovator (the more R&D-intensive firm) and imitator (the less R&D-intensive firm) are endogenously determined. That is, a firm always either spends less than its rival so as to free-ride on the latter's R&D investment through spillovers, or spends more if the other firm's investment is too low in order to benefit from a competitive advantage over its rival in the product market. This outcome produces asymmetries in terms of the unit cost structure in the product market competition, and thus unequal market shares. This conclusion establishes a simple link between the nature of the R&D process in an industry—including the associated spillover—and the emergence of inter-firm heterogeneity in that industry.

As we shall see below, it turns out to be more convenient to examine some of the economic issues considered in this paper with a deterministic spillover process than with its stochastic analog (proposed by AW). One aspect of this choice is motivated by the ease of comparison with the deterministic multiway spillover processes typically used in the literature (as in Amir, 2000; d'Aspremont & Jacquemin, 1988; Kamien *et al.*, 1992). Another difference between this setting and the stochastic version in AW is that endogenous heterogeneity of firms in terms of R&D and final unit costs holds with certainty in the present setting, but only with positive probability in the AW model.

In the second part of the paper, we study R&D cooperation among firms by means of the formation of a joint lab, thereby allowing firms to jointly appropriate the outcome of R&D investments, while sharing the associated costs equally, as in Amir (2000). Kamien *et al.* (1992) have shown that, when the spillover process is multidirectional and deterministic, cooperating through a joint lab is superior to R&D competition in terms of levels of investments, industry profit, and consumer surplus.[4] In the context of one-way stochastic spillovers, AW find that, under R&D competition, the innovator sometimes invests more in R&D than the joint lab, and the industry's total profit is sometimes higher than under the joint lab. Clearly, because spillovers vanish under this type of cooperation, the same results obtain with deterministic one-way R&D spillovers.[5]

In the third and most important part of the paper, we consider a benevolent central planner with a second-best mandate, that is, one that can decide on R&D investments without intervening as far as market conduct is concerned (as

[2] That spillovers are an important aspect of firms' overall business strategy is well documented (see, e.g., Billand, Bravard, Chakrabarti, & Sarangi, 2016, for an overview of the related literature). In addition, there are multiple channels through which spillovers might flow, including in trade-related contexts (see, e.g., Ferrier, Reyes, & Zhu, 2016).

[3] For related settings, see also Jin and Troege (2006), Hinloopen (1997, 2000), Martin (2002), and Tesoriere (2008).

[4] More precisely, these authors considered a cartelized joint venture defined as an R&D cartel (i.e, firms choose R&D levels to maximize their total Cournot profits) wherein firms internally set the spillover parameter to its maximal value of 1. Amir (2000) shows that this cooperation scenario is equivalent to a joint lab.

[5] The literature on R&D cooperation has more recently been extended to other areas of economics, including environmental innovation (McDonald & Poyago-Theotoky, 2017), and the organization of the firm (Chalioti, 2015).

in Suzumura, 1992). We consider two different scenarios, one in which the planner is subject to the political constraint of equal treatment of the firms, and one in which the planner is free from such constraints. While the second-best optimal symmetric investments coincide with those of the joint lab in the first scenario, social welfare achieved under the joint lab is superior because R&D costs are shared among firms. Therefore, the joint lab emerges as a desirable way to implement a constrained second-best optimal scenario without actual intervention by a social planner (and with built-in avoidance of R&D duplication costs).

Furthermore, since imposing symmetric R&D investments yields symmetric final unit costs in the product market competition, social welfare under R&D competition may dominate that under the symmetry-constrained central planner; in fact, this happens when R&D costs are low enough. Intuitively, this is not that surprising since social welfare tends to be higher when firms are asymmetric in terms of unit costs (see Salant & Shaffer, 1998, 1999). In fact, as the latter studies brought to the fore, welfare maximization often entails endogenous discriminatory treatment of firms even under the standard multi-directional spillover structure. Thus, one important motivation for the welfare part of the present paper is precisely that this endogenous discriminatory outcome will be even more significant under a unidirectional spillover structure.

In this respect, it is obviously of interest to get a handle on the extent of welfare loss incurred by society as a result of the politically motivated constraint of equal treatment of firms in regulation. Relaxing the assumption that the central planner imposes equal treatment of firms, we find that social welfare induced by the second-best welfare maximizing asymmetric R&D investments dominate that of the joint lab if either the spillover parameter or the cost of performing R&D are low enough. Therefore, the well known result that the market typically delivers lower levels of R&D than a (second-best) social planner continues to hold in our setting, despite the resulting asymmetry among firms.[6] Finally, we show that the efficiency loss due to equal treatment increases with the size of the spillover parameter, and may amount to a maximal level of about 45% in relative terms. We argue that this is a surprisingly high and significant loss, and that, broadly speaking, market regulators may be well advised to take this into account when conceiving regulatory schemes.

The rest of the paper is organized as follows. Section 2 describes the basic noncooperative R&D model and the associated assumptions. Section 3 characterizes the equilibrium under R&D competition. Section 4 studies the effects of R&D cooperation by means of a joint lab. An extensive second-best welfare analysis is provided in Section 5, including a comparison with the non cooperative scenario and with the joint lab. Concluding remarks are provided in Section 6. All the proofs (in the form of brief calculations) are provided in the appendix.

2 | THE MODEL

The basic model is a standard two-stage duopoly consisting of a process R&D choice in the first stage and subsequent Cournot competition in the second stage in the tradition of the literature following Katz (1986) and d'Aspremont and Jacquemin (1988). However, although R&D is still subject to involuntary spillovers, these will be taken to one-way or unidirectional in the present study, following Amir and Wooders (2000), henceforth AW.

Formally, consider an industry with two firms producing a homogenous good with the same initial unit cost c, playing the following two-stage game. In the first stage, firms simultaneously choose their autonomous cost reduction level x_1 and x_2, with $x_i \in [0, c]$, $i = 1, 2$. The R&D cost to firm i associated with the cost reduction x_i is

$$C(x_i) = \frac{\gamma}{2} x_i^2, \quad i = 1, 2.$$

We assume, following AW, that spillovers are unidirectional, or in other words that know-how flows only from the more R&D-intensive firm (called the innovator) to its rival (the imitator). However, contrary to AW, we assume that

[6] For instance, Burr, Knauff, and Stepanova (2013) provide some insight into the well known wedge between private and social levels of R&D. See also Stepanova and Tesoriere (2011) and Amir, Halmenschlager, and Knauff (2017).

the spillover process is deterministic; namely, if autonomous cost reductions are x_1 and x_2 with, say, $x_1 \geq x_2$, then the effective or final cost reductions are

$$X_1 = x_1 \quad \text{and} \quad X_2 = x_2 + \beta(x_1 - x_2),$$

where the parameter $\beta \in [0, 1]$, called the spillover parameter, is the fraction of the difference in cost reductions that spills over to firm 2 (with certainty).[7] Thus, the imitator ends up with its own cost reduction plus a fraction of the innovator's lead. This is a natural definition of spillovers in settings where the R&D process is one-dimensional, reflecting in particular that firm 1 has nothing to possibly learn from firm 2.

This deterministic spillover process may be seen as the certainty-equivalent version of the stochastic spillover process introduced by AW. Both spillover processes are a reflection of the R&D process itself being a one-dimensional process, that is, a well defined sequence of hurdles or tests that firms may pursue in their search for discovery.[8]

In the second stage, upon observing the new unit costs firms compete in the product market by choosing quantities, facing a linear inverse demand

$$P(q_1 + q_2) = a - (q_1 + q_2).$$

A pure strategy for firm *i* is thus a pair (x_i, q_i), where $x_i \in [0, c]$ and $q_i : [0, c]^2 \to \mathbb{R}_+$. Throughout, we use the standard concept of subgame perfect equilibrium.

We assume that demand is high enough relative to the initial unit cost to ensure that the second-stage game admits a unique pure strategy Nash equilibrium (PSNE) where both firms are active in the product market for all possible R&D levels that they may undertake, that is,

Assumption 1. $a > 2c$.

Cournot equilibrium profit of firm *i* in the second stage, given the actual unit costs c_i, c_j, is thus given by $\Pi(c_i, c_j) = (a - 2c_i + c_j)^2/9$. Firms' *net* profits F_1, F_2, defined as the difference between the second-stage profit and the first-stage R&D investment, can then be expressed as functions of the autonomous cost reductions x_1 and x_2. Since the game is symmetric, we have that $F_1(x_1, x_2) = F_2(x_2, x_1)$. Therefore, throughout the paper, we omit the subscripts and write $F(x_i, x_j)$ to denote the net profit of firm *i*, where

$$F(x_i, x_j) = \begin{cases} \frac{1}{9}[a - c + x_i(2-\beta) - x_j(1-\beta)]^2 - \frac{\gamma}{2}x_i^2 & \hat{=} U(x_i, x_j) \quad \text{if } x_i \geq x_j \\ \frac{1}{9}[a - c + 2x_i(1-\beta) + x_j(2\beta - 1)]^2 - \frac{\gamma}{2}x_i^2 & \hat{=} L(x_i, x_j) \quad \text{if } x_i \leq x_j. \end{cases} \tag{1}$$

One can easily check that F is globally continuous, concave in the two triangles above and below the diagonal, but has a concavity-destroying kink along the diagonal. Furthermore, for $\beta \leq \frac{1}{2}$, both U and L are submodular in (x_i, x_j), that is, $\frac{\partial^2 U(x_i, x_j)}{\partial x_i \partial x_j} < 0$ and $\frac{\partial^2 L(x_i, x_j)}{\partial x_i \partial x_j} < 0$. On the other hand, for $\beta > \frac{1}{2}$, U is submodular but L is supermodular in (x_i, x_j), that is, $\frac{\partial^2 U(x_i, x_j)}{\partial x_i \partial x_j} < 0$ and $\frac{\partial^2 L(x_i, x_j)}{\partial x_i \partial x_j} > 0$.

Furthermore, we assume the following:

Assumption 2. $9\gamma > 2(2 - \beta)^2$.

Assumption 3. $9\gamma > 4\frac{a}{c}(1 - \beta)$.

[7] An analogous pair of expressions for the final cost reductions holds for the case $x_1 \leq x_2$, and is thus omitted.

[8] More precisely, the process need not be uni dimensional as long as there is a natural sequence of search steps that all firms would undertake. This is quite distinct from the multi dimensional heuristic proposed by Kamien *et al.* (1992) as an appropriate R&D process corresponding to the multiway spillover process that is widely adopted in the literature, starting with Spence (1984). Naturally, different industries will be better approximated by one or the other of these two categories of spillover process.

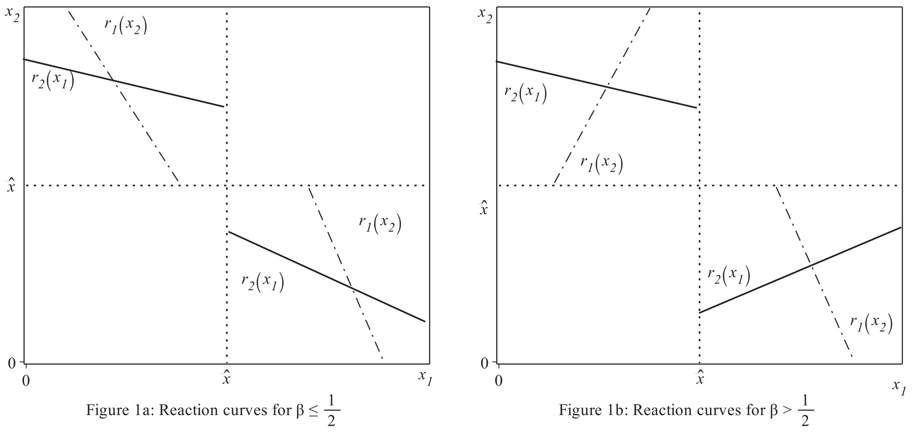

FIGURE 1 Reaction curves for different values of β

Close variants of these assumptions are quite standard in the R&D literature. Assumption 2 guarantees that U and L are strictly concave with respect to own R&D level, and may thus be thought of as a global second-order condition. Assumption 3 ensures that firm i's reaction function is interior, or that it satisfies $r_i(c) < c$, where $r_i(x_j) \in \text{argmax}\{F(x_i, x_j) : x_i \in [0, c]\}$.

3 | THE NONCOOPERATIVE EQUILIBRIUM

In this section, we analyze the subgame-perfect equilibria of the two-stage game, to be referred to as Case N (for noncooperative scenario). Equivalently, we analyze the Nash equilibria of the game in R&D choices, given the unique Cournot equilibrium in the second stage (with payoffs given by (1)).

Under Assumptions 2 and 3, one can derive the reaction function of, say, firm i as

$$r_i(x_j) = \begin{cases} 2\frac{(2-\beta)(a-c+x_j(\beta-1))}{9\gamma-2(\beta-2)^2} & \text{if } x_i \geq x_j \\ 4\frac{(1-\beta)(a-c+x_j(2\beta-1))}{9\gamma-8(\beta-1)^2} & \text{if } x_i \leq x_j, \end{cases} \tag{2}$$

and because the game is symmetric, we have $r_i(x_j) = r_j(x_i)$.

Before characterizing the equilibrium investments of the first-stage R&D game, our first result sheds light on a key feature of the model, that firms' reaction functions cannot be continuous.

Lemma 1. *The reaction functions admit a unique downward jump that skips over the* 45° *line.*

Figure 1a (resp. 1b) depicts firms' reaction curves for $\beta \leq \frac{1}{2}$ (resp. $\beta > \frac{1}{2}$). As was previously mentioned, the upper payoff function U is globally submodular in own and rival's decisions so that it gives rise to a reaction function segment that shifts down as rival's investment increases. As for the lower payoff function L, it is also submodular in own and rival's decision for $\beta \leq \frac{1}{2}$ (thus reflecting strategic substitutes), but supermodular for $\beta > \frac{1}{2}$, so that its reaction function segment shifts up (thus reflecting strategic complements) as rival's investment increases for this range of the spillover parameter.[9]

[9] In contrast to our model, each player's payoff function in AW is instead globally submodular in (x_i, x_j) so that reaction curves have the same shape as those depicted in Figure 1a.

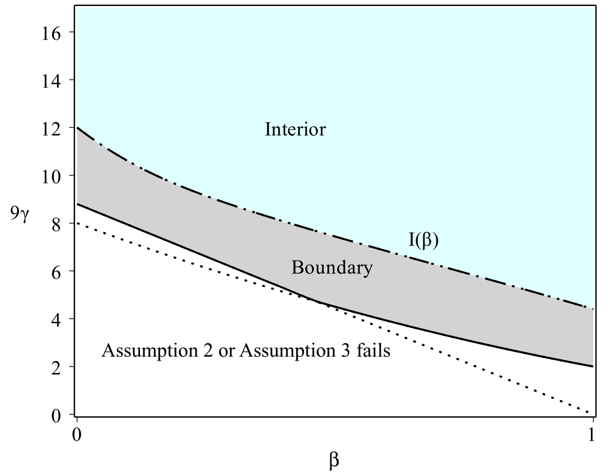

FIGURE 2 Type of equilibrium

Given the firms' reaction functions, straightforward computations establish that reaction curves cross at $(\bar{x}, \underline{x})$ and $(\underline{x}, \bar{x})$, where

$$\begin{cases} \bar{x} = \frac{1}{D_N} 2(2-\beta)[3\gamma - 4(1-\beta)^2](a-c) \text{ and} \\ \underline{x} = \frac{1}{D_N} 4(1-\beta)[3\gamma - 2(1-\beta)(2-\beta)](a-c), \end{cases}$$

where

$$D_N \triangleq 27\gamma^2 - 6\gamma(5\beta^2 - 12\beta + 8) + 8(2-\beta)(1-\beta)^2.$$

It is easy to verify that $\bar{x} > \underline{x}$ for any $\beta \in (0, 1)$.

We need one further assumption on the parameters of the model (for interiority).

Assumption 4.

$$9\gamma > I(\beta) \triangleq \left(\frac{a}{c} - 1\right)(2-\beta) + (5\beta^2 - 12\beta + 8) + \\ + \sqrt{\left(\left(\frac{a}{c} - 1\right)(2-\beta) + (5\beta^2 - 12\beta + 8)\right)^2 - 24\frac{a}{c}(2-\beta)(1-\beta)^2}.$$

We begin with a characterization of the set of PSNE in the R&D game with payoffs given in (1), and thus of subgame-perfect equilibria of the two-stage game.

Proposition 1. *Under Assumptions 1–4, the R&D game admits a unique pair of PSNE of the form* $(\bar{x}, \underline{x})$ *and* $(\underline{x}, \bar{x})$.

Thus, although firms are *ex ante* identical, only asymmetric equilibrium pairs of R&D investments prevail. This gives rise endogenously to a high R&D firm (called the innovator) and a low R&D firm (called the imitator).

As in the stochastic version of the model, the equilibrium levels of R&D investments are asymmetric due to the non-concavity of the net profit function F along the 45° line. By Lemma 1, reaction curves jump downward over the diagonal at $\hat{x}$ as indicated in Figure 1 so that, in equilibrium, a firm will always either spend less than its rival so as to free-ride on the latter's R&D investment through spillovers, or spend more if the other firm's investment is too low in order to benefit from a competitive advantage over its rival in the product market. Notice that Assumption 4 ensures that the two equilibrium pairs $(\bar{x}, \underline{x})$ and $(\underline{x}, \bar{x})$ are interior solutions. Instead, if Assumptions 1 through 3 are satisfied, but Assumption 4 is not, we have a boundary equilibrium of the form $(\bar{x}^B, \underline{x}^B)$ and $(\underline{x}^B, \bar{x}^B)$, where $\bar{x}^B = c$ and $\underline{x}^B = 4\frac{(a-2c(1-\beta))(1-\beta)}{9\gamma - 8(\beta-1)^2}$. Figure 2 graphs Assumptions 2 through 4 in the parameter space $(\beta, 9\gamma)$ and shows whether an interior or a boundary equilibrium prevails.

As long as $\beta > 0$, endogenous heterogeneity of firms will prevail with certainty in the present model. The one-dimensional nature of the R&D process gives rise naturally to one-way spillovers, which in turn provide incentives for firms to break off into an innovator and an imitator.[10] While a similar outcome prevails on average in the AW model, endogenous heterogeneity of firms materializes only with probability $(1-\beta)$, that is, when no spillover takes place *ex post*.

Two special cases of the spillover parameter are worth highlighting. When $\beta = 1$, R&D is a pure public good, and the equilibrium autonomous and effective R&D levels, which reflect complete free-riding on the part of the follower (firm 2) as one would expect, are[11]

$$\overline{x} = \frac{4(a-c)}{9\gamma-2},\ \underline{x} = 0 \quad \text{and} \quad X_1 = X_2 = \frac{4(a-c)}{9\gamma-2}.$$

When $\beta = 0$, R&D is a pure private good, and the equilibrium autonomous and effective R&D levels reduce to

$$\overline{x} = \underline{x} = X_1 = X_2 = \frac{4(a-c)}{9\gamma-4}.$$

4 | R&D COOPERATION

In this section, we examine R&D cooperation by means of a joint lab, which allows firms to jointly appropriate the outcome of R&D investments in one and the same lab, while equally sharing the associated cost. This cooperation scenario was introduced in Amir (2000) both as a maximal R&D cooperation scenario and as a useful benchmark due to the absence of any spillover effects. This case will be referred to as Case J.

Recall that in models featuring the standard multidirectional spillover process with input spillovers (as in Kamien *et al.*, 1992; Spence, 1984), the so-called R&D cartel with spillover parameter internally increased to its maximal value of 1 is equivalent to a joint lab (as shown in Amir, 2000). This equivalence justifies viewing the joint lab as a maximal R&D cooperation scenario. In addition, this cooperation scenario delivers superior overall performance; it is shown in Kamien *et al.* (1992) to dominate the other commonly used scenarios in terms of resulting firms' propensities for R&D, firms' profit, and consumer surplus (and thus also social welfare).

In line with its superior performance, the interest in this form of cooperation in the present study will be seen to also lie in the fact that it coincides with a constrained version of the second-best outcome.

Under this scenario, the joint lab chooses a level of R&D that maximizes the sum of firms' profits, net of the (shared) R&D cost. Thus, the problem of the joint lab is

$$\max_{x\in[0,c]} \left\{ \frac{2}{9}(a-c+x)^2 - \frac{\gamma}{2}x^2 \right\}.$$

The maximization yields the following per firm optimal level of investment:

$$x_J = \begin{cases} 4\frac{a-c}{9\gamma-4} & \text{if} 9\gamma > 4\frac{a}{c} \\ c & \text{otherwise.} \end{cases}$$

Figure 3 shows whether an interior or a boundary equilibrium prevails under the joint lab.

The next proposition provides a comparison of the joint lab's R&D investment with those of the noncooperative game (as depicted on Figure 3).

Proposition 2. *Under Assumptions 1–4, the comparison of the equilibrium R&D levels in cases J and N is as follows:*

[10] As such, the present paper joins a recent trend of research in applied theoretical economics dealing with the endogenous emergence of asymmetric outcomes pertaining to *ex ante* identical agents. This is generally referred to as symmetry-breaking (a term borrowed from theoretical physics), or endogenous heterogeneity. See, *inter alia*, Matsuyama (2002), Amir, Garcia and Knauff (2010), Basu, Basu and Cordella (2016), Yazici (2016), Acemoglu, Robinson, and Verdier (2017), and Chatterjee (2017).

[11] Indeed, conditional on being a follower, a firm has a dominant strategy of doing no R&D, as reflected by a reaction curve identically equal to 0 in (2).

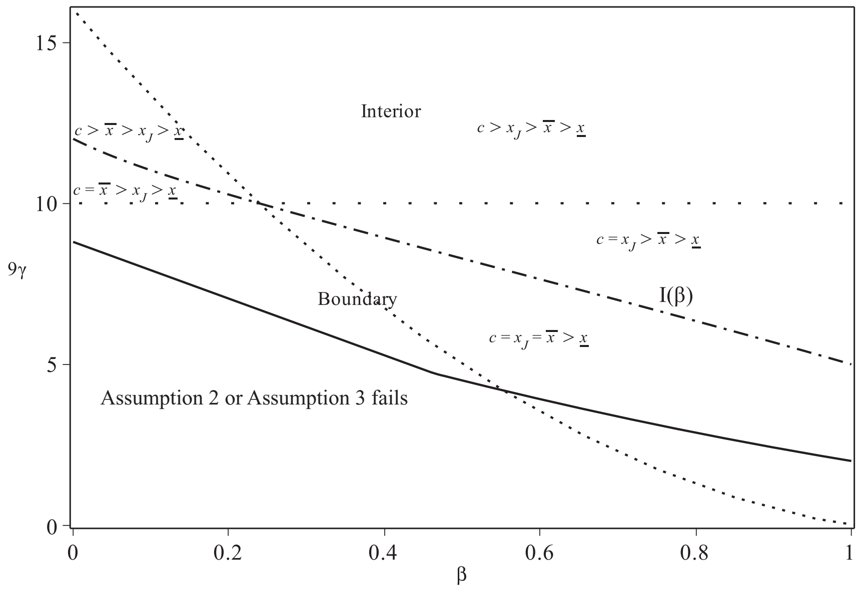

FIGURE 3 Comparison of equilibrium R&D levels in cases N and J

(i) $x_J = \overline{x} = \underline{x} = \frac{4(a-c)}{9\gamma-4}$ *if and only if* $\beta = 0$.

(ii) $x_J < \bar{x}$ *if* $9\gamma < 4(1-\beta)(4-3\beta)$, *and* $x_J > \bar{x}$ *otherwise.*

(iii) $x_J > \underline{x} + \beta(\bar{x} - \underline{x})$.

(iv) *If both* $(\overline{x}, \underline{x})$ *and* x_J *are interior, then total effective cost reductions under the joint lab (case J) dominate those of the noncooperative setting (case N).*

The first part of the result captures the fact that, without any spillovers, the firms not only undertake the same level of R&D at equilibrium, but also that they actually undertake the same level as they would in a joint lab.

The second part says that the innovator invests more in R&D than the joint lab if the spillover parameter and/or the R&D costs are low enough. Intuitively, in the noncooperative setting, the prospect of efficiency gains when competing in the product market with a weaker rival boosts R&D investments in the first stage if these two conditions are satisfied. Conversely, its incentives to exert R&D effort are undermined if either the associated cost is large or the fraction of its cost reduction that spills over the imitator is high. Consequently, in this case, the joint lab reaches a higher level of cost reduction by both splitting the cost of undertaking R&D among firms and suppressing the free-rider issue.

Not surprisingly, the level of R&D performed by the imitator in the noncooperative case is instead strictly lower than the joint lab's optimal cost reduction for any R&D cost and any spillover rate because R&D competition leaves scope for free riding over the innovator's investment due to the existence of spillovers.

Finally, for interior solutions, the total effective cost reduction achieved by means of cooperation via a joint lab is greater than in the noncooperative case. This last finding is in line with past results in the literature on the joint lab's superiority in terms of the resulting propensity for R&D (Amir, 2000; Kamien *et al.*, 1992).

Next, we examine the impact of R&D cooperation on firms' equilibrium profit. Equilibrium per-firm profit under a joint lab is given by

$$\tilde{F}(x_J) = \begin{cases} \frac{\gamma(a-c)^2}{9\gamma-4} & \text{if } 9\gamma > 4\frac{a}{c} \\ \frac{a^2}{9} - \frac{\gamma c^2}{4} & \text{otherwise.} \end{cases}$$

The following table provides a full comparison of the innovator's equilibrium profit for interior and boundary equilibria in both settings.

		R&D cooperation through a joint lab	
		Interior ($9\gamma > 4\frac{a}{c}$)	Boundary ($9\gamma < 4\frac{a}{c}$)
R&D competition	Interior ($9\gamma > I(\beta)$)	$\tilde{F}(x_J) > F(\bar{x}, \underline{x})$	$\tilde{F}(c) > F(\bar{x}, \underline{x})$
	Boundary ($9\gamma \leq I(\beta)$)	$\tilde{F}(x_J) \gtreqless F(\bar{x}^B, \underline{x}^B)$	$\tilde{F}(c) > F(\bar{x}^B, \underline{x}^B)$

Observe first that the innovator is strictly better off cooperating with its rival whenever the interior equilibrium would otherwise prevail in the noncooperative setting since the joint lab allows to share both the cost and the results of R&D investments, while leaving no role for spillovers. The superiority of cooperation is nonetheless jeopardized whenever $4\frac{a}{c} < 9\gamma \leq I(\beta)$. In this region of parameters, R&D competition enables the innovator to stand out from its rival in terms of efficiency gains in the product market, so that its higher profit at the competition stage outweighs both the cost of undertaking R&D and the losses associated with a lower appropriability of its investment. Notably, the innovator strictly prefers to incur substantial R&D expenditures the lower the spillover parameter, so that asymmetries in terms of unit costs are exacerbated at the competition stage. For instance, letting $a = 2.2$, $c = 1$, $\beta = 0.1$, and $\gamma = 1.16$, we have that $\tilde{F}(x_J) = 0.259 < 0.274 = F(\bar{x}^B, \underline{x}^B)$.

Likewise, the next table compares the imitator's equilibrium profit under both regimes.

		R&D cooperation through a joint lab	
		Interior ($9\gamma > 4\frac{a}{c}$)	Boundary ($9\gamma < 4\frac{a}{c}$)
R&D competition	Interior ($9\gamma > I(\beta)$)	$\tilde{F}(x_J) > F(\underline{x}, \bar{x})$	$\tilde{F}(c) \gtreqless F(\underline{x}, \bar{x})$
	Boundary ($9\gamma \leq I(\beta)$)	$\tilde{F}(x_J) > F(\underline{x}^B, \bar{x}^B)$	$\tilde{F}(c) > F(\underline{x}^B, \bar{x}^B)$

The imitator strictly prefers cooperating with the innovator unless $I(\beta) < 9\gamma < 4\frac{a}{c}$, in which case R&D competition may be superior as it enables the imitator to freely benefit from the innovator's investment through spillovers. More specifically, free riding becomes more profitable than sharing the cost of undertaking R&D with the innovator for large values of the spillover parameter. To see this, let $a = 5$, $c = 1$, $\beta = 0.95$, and $\gamma = 1.38$. Then, we have that $\tilde{F}(c) = 2.43$, while $F(\underline{x}, \bar{x}) = 2.5$.

Even though firms inherit the same cost structure under a joint lab, thereby dissipating firms' total profit in the product market, cooperation through a joint lab allows firms to share R&D costs, thereby avoiding the inefficiencies associated with the free-rider issue inherent to the noncooperative setting. The next result establishes that the latter effect dominates the former, thereby making the industry strictly better off when cooperating through a joint lab.

Proposition 3. *$2\tilde{F}(c) \geq F(\bar{x}, \underline{x}) + F(\underline{x}, \bar{x})$ and $2\tilde{F}(x_J) \geq F(\bar{x}^B, \underline{x}^B) + F(\underline{x}^B, \bar{x}^B)$.*

As a brief conclusion for this section, it may be said that, while the endogenous asymmetry can reverse some of the established conclusions on the superiority of the joint lab in the literature, these conclusions can be restored when considering aggregate performance.

5 | WELFARE ANALYSIS

In this section, we consider a benevolent central planner with a second-best mandate, that is, one that is endowed with the authority to decide on R&D investments but has no control or influence over the firms' market conduct once R&D levels have been selected. We first examine the case where the planner is constrained to impose symmetric R&D

expenditures across firms (i.e, to satisfy the principle of equal treatment of equals). We shall refer to this planner's scenario as Case P_S (for second-best planning under symmetric treatment of firms).[12]

We then consider the case where the social planner is unconstrained in its choices of R&D levels and can thus exploit the benefits of asymmetric choices. We abbreviate this planner's scenario as Case P_A (for second-best planning under possible asymmetry). Finally, we compare the two different planning solutions and examine the social costs of imposing equal treatment among firms.

Assume without loss of generality that $x_1 > x_2$, and define social surplus (welfare) in the usual way as the sum of firms' profit and consumer surplus, that is, we have

$$S(x_1, x_2, q_1, q_2) = (q_1 + q_2)\left(a - \frac{(q_1 + q_2)}{2}\right) - (c - x_1)q_1 - (c - (x_2 + \beta(x_1 - x_2)))q_2 - \frac{\gamma}{2}\left(x_1^2 + x_2^2\right).$$

Given the Cournot equilibrium in the second stage of the game, one can write social welfare as a function of the R&D (first-period) decisions by substituting the Cournot outputs into $S(x_1, x_2, q_1, q_2)$. This yields

$$\begin{aligned} W(x_1, x_2) = &\frac{1}{18}\left[8(a - c)\left((a - c) + x_1(1 + \beta) + x_2(1 - \beta)\right)\right. \\ &\left. - x_1^2\left(9\gamma + 14\beta - 11\beta^2 - 11\right) - x_2^2\left(9\gamma - 11(1 - \beta)^2\right) + 2x_1x_2(11\beta - 7)(1 - \beta)\right]. \end{aligned} \quad (3)$$

5.1 | Symmetric second-best planner's solution

In this subsection, the planner is constrained to select symmetric R&D expenditures for the two identical firms, and thus to satisfy the principle of equal treatment of equals when engaging in any sort of regulation of firms. The main finding here is that the planner's solution yields the same R&D for each firm as a joint lab.

The problem of the central planner is thus (see (3))

$$\max_{(x_1, x_2) \in [0, c]^2} \{W(x_1, x_2) : x_1 = x_2\}.$$

Upon a simple computation, the optimal symmetric per firm investment level and the corresponding symmetric-optimal second-best welfare are given by

$$x_s = \frac{4(a - c)}{9\gamma - 4} \quad \text{and} \quad W\left(x_s, x_s\right) = \frac{4\gamma(a - c)^2}{9\gamma - 4}.$$

As seen by simple inspection, the symmetry-constrained socially optimal level of R&D coincides with the optimal R&D level of the joint lab, that is, $x_s = x_J$, and therefore

$$x_s = \overline{x} = \underline{x} = \frac{4(a - c)}{9\gamma - 4} \quad \text{if and only if} \quad \beta = 0.$$

This coincidence of R&D levels means that the joint lab is a socially optimal form of R&D cooperation under the equal treatment restriction. Furthermore, noncooperative R&D yields a second-best symmetry-constrained socially optimal level of R&D as long as R&D spillovers are fully absent. The latter requirement is quite unrealistic, since spillovers are typically considered as an unavoidable characteristic of the technological environment in an industry.[13]

Another direct implication of this outcome is that a joint lab emerges as one possible and practical way to implement a symmetric socially optimal scenario without involving a central planner at all.

[12] It is worth stressing that this principle is widely taken for granted in the formulation of public policy. As such, it is generally not even a subject of debate, although, as we shall see, this is not necessarily in society's interest in the present context.

[13] Naturally, the level of spillovers may be influenced by location patterns, patent policy, and other factors. Nevertheless, it is unrealistic to assume that they may be driven down all the way to zero.

Furthermore, because the joint lab avoids the duplication of R&D costs by definition, it leads to a welfare level W_J that is clearly strictly higher then the symmetric-optimal second-best welfare, that is, $W_J > W(x_s, x_s)$.[14] A brief calculation shows that

$$W_J = \frac{4\gamma(9\gamma - 2)(a-c)^2}{(9\gamma - 4)^2},$$

The fact that $x_s = x_J$ yields the following from the results on the joint lab.

Proposition 4. *The comparison of the equilibrium R&D levels in cases P_S and N is as follows:*

- **(i)** $x_s = x_J = \bar{x} = \underline{x} = \frac{4(a-c)}{9\gamma-4}$ *if and only if* $\beta = 0$.
- **(ii)** $x_s < \bar{x}$ *if* $9\gamma < 4(1-\beta)(4-3\beta)$, *and* $x_s > \bar{x}$ *otherwise.*
- **(iii)** $x_s > \underline{x}$ *always.*

Again, the case of no spillovers yields an exceptional outcome worth highlighting. Not only does the noncooperative solution coincide with the joint lab, it also yields a symmetry-constrained second-best socially optimal level of R&D. The direct implication of this simple observation is obvious yet quite important: With no spillovers, the market solution is second-best efficient (albeit in a constrained manner), so *laissez-faire*, as opposed to both intervention or a joint lab, is the way to go.

While the unconstrained social planner inherits the incentive to create a high R&D firm and a low R&D firm, just like firms in Case N, the planner would generate a smaller spread between the firms when R&D costs are low (or γ small), and a higher level of R&D for both firms when R&D costs are high. The latter outcome is of course the standard implication of social planning for R&D, the aim being to correct for the market's tendency to supply too little R&D, due to the well known and documented gap between private and social returns to R&D (see, e.g., Bernstein & Nadiri, 1988; Griliches, 1995). On the other hand, it is certainly noteworthy that, with low R&D costs, the market outcome leads to more R&D for the innovator than the socially optimal solution.

Whether symmetric-optimal welfare $W(x_s, x_s)$ is superior to that of the noncooperative setting instead depends on the magnitude of R&D costs. Two conflicting effects need to be considered. On the one hand, imposing symmetric R&D investments implies that firms face the same unit cost when competing in the product market, which in turn leads to total profit dissipation. On the other hand, because $x_s = x_J$, total effective cost reductions are higher in the symmetric planner's solution (see Proposition 2). It follows that consumers benefit from a lower price under the latter solution. The following result characterizes regions of parameters for which either effect dominates.

Proposition 5. *The ranking of welfare levels under Cases N, J, and P_S is as follows:*

$$W_J \geq \begin{cases} W(\bar{x}, \underline{x}) > W\left(x_s, x_s\right) & \text{if } 9\gamma < K \\ W\left(x_s, x_s\right) > W(\bar{x}, \underline{x}) & \text{otherwise,} \end{cases}$$

where

$$K = \frac{1}{2}(43\beta^2 - 102\beta + 55) + \frac{1}{2}\sqrt{1057 - 4212\beta + 5870\beta^2 - 3396\beta^3 + 697\beta^4}.$$

Not surprisingly, total welfare when firms cooperate through a joint lab exceeds that of the noncooperative setting. Indeed, both the innovator and the imitator get a strictly higher profit when cooperating (cf. Section 4). Moreover, since the second-best welfare maximizing symmetric R&D investments coincide with those of the joint lab, Proposition 5 applies. Namely, total effective cost reductions are higher than those achieved in the noncooperative setting so that aggregate production costs are lower whenever a central planner intervenes in the R&D game. It directly follows that firms charge a lower price, and that consumer surplus is higher, that is, $CS(x_s, x_s) \geq CS(\bar{x}, \underline{x})$.

[14] More precisely, $W_J = W(x_s, x_s) - \frac{\gamma x_s^2}{4}$. Note that W_J cannot be expressed via the function $W(\cdot, \cdot)$.

Recall that Kamien *et al.* (1992) demonstrated that, in the analogous model but with two-way spillovers, the joint lab (or the cartelized R&D joint venture as they described it in an equivalent manner) is superior in terms of propensity for R&D and social welfare to the other three scenarios examined in that paper. Here, we show that, with one-way spillovers, the joint lab actually yields the socially optimal level of R&D subject to the equal treatment restriction.

The key conclusion of this part is that a joint lab may be regarded as a simple and noninterventionist manner of actually implementing a second-best socially optimal outcome for a duopoly with one-way spillovers. Indeed, while second-best planning is often taken as a useful benchmark for policy analysis, a joint lab represents an actual institution that can not only lead to the social levels of R&D, but also avoid the duplication costs in carrying out the R&D.

Next, we relax the assumption that the social planner imposes equal treatment across firms.

5.2 | Asymmetric second-best planner's solution

In this part, the problem of the social planner is now to choose a pair of (possibly asymmetric) R&D investments that maximizes total welfare, as given by Equation 3, that is,

$$(\overline{x}_a, \underline{x}_a) \in \underset{(x_1,x_2)\in[0,c]^2}{\operatorname{argmax}} W(x_1, x_2). \tag{4}$$

Intuitively, one would expect the global argmax of social welfare to be asymmetric, as a result of the well known fact that Cournot equilibrium industry profit is convex in firms unit costs. In other words, industry profit tends to be higher when firms are asymmetric in terms of unit costs, and this property is inherited by social welfare (see, e.g., Salant & Shaffer, 1998, 1999; Soubeyran & Van Long, 1999).

Indeed, this intuition is confirmed by the solution, as it may be easily verified that the optimal investment levels are given by

$$\overline{x}_a = \frac{4}{D_a}[\gamma(\beta+1) - 2(1-\beta)^2](a-c) \quad \text{and} \quad \underline{x}_a = \frac{4}{D_a}[\gamma - 2(1-\beta)](1-\beta)(a-c),$$

where

$$D_a \triangleq 9\gamma^2 - 2\gamma(11\beta^2 - 18\beta + 11) + 8(1-\beta)^2.$$

It is easy to check that this solution is always asymmetric for any nonzero value of β, that is, that

$$\overline{x}_a \geq \underline{x}_a \text{ with equality if and only if } \beta = 0.$$

Therefore, in industries with nonzero one-way spillovers, the social planner always faces a clear incentive for unequal treatment of regulated firms, even when these are *ex ante* symmetrical.[15]

Two special cases of the spillover parameter are worth reporting. When $\beta = 1$, R&D is a pure public good, and the second-best autonomous and effective R&D levels, reflecting the fact that the planner takes full advantage of the perfect spillovers for the follower (firm 2), are

$$(\overline{x}_a, \underline{x}_a) = \left(\frac{8(a-c)}{9\gamma-8}, 0\right) \quad \text{and} \quad X_1 = X_2 = \frac{8(a-c)}{9\gamma-2}.$$

[15] A lengthy computation shows that $0 < x_2^W < x_1^W < c$ if $9\gamma > 18(1-\beta)$ and $9\gamma > Z_1$, where

$$Z_1 = 2\frac{a}{c}(1+\beta) - (11\beta - 9)(1-\beta) + \\ + \frac{1}{c}\sqrt{(11\beta-9)^2(\beta-1)^2 + 4(\frac{a}{c})^2(\beta+1)^2 + 4\frac{a}{c}(\beta-1)\left(11\beta^2 - 16\beta + 9\right)}.$$

When $\beta = 0$, R&D is a pure private good, and the autonomous and effective R&D levels reduce to

$$\bar{x} = \underline{x} = X_1 = X_2 = \frac{4(a-c)}{9\gamma - 4}.$$

The corresponding optimal level of social welfare for any $\beta \in [0, 1]$ is

$$W(\bar{x}_a, \underline{x}_a) = \frac{4}{D_a}\gamma[\gamma - 2(1-\beta)^2](a-c)^2.$$

Since symmetric choices of R&D levels are one option that the social planner has in the optimization problem (4), it follows that $W(\bar{x}_a, \underline{x}_W) > W(x_W, x_W)$, as is easily verified by direct calculation. Nevertheless, despite its suboptimality, the constrained-symmetric solution may well be of substantial real-life interest, since implementing an asymmetric solution on *a priori* identical firms is likely to be politically infeasible. It would be akin to forging a national champion and a weak firm out of two equally efficient firms.

Our next result compares the second-best welfare-maximizing asymmetric R&D investments with those of the non-cooperative setting, as well as the associated effective cost reductions.

Proposition 6. *The second-best welfare maximizing asymmetric R&D investments satisfy*

- **(i)** $\bar{x}_a > \bar{x}$ *and* $\underline{x}_a > \underline{x}$ *if* $\beta > \frac{2}{3}$, *while* $\underline{x}_a < \underline{x}$ *if both* $\beta \le \frac{2}{3}$ *and* $9\gamma > \frac{2(1-\beta)(23\beta-11-11\beta^2)}{(3\beta-2)}$.
- **(ii)** $(1+\beta)\bar{x}_a + (1-\beta)\underline{x}_a > (1+\beta)\bar{x} + (1-\beta)\underline{x}$.

For part (i), it is noteworthy that for small spillover rates, the social planner would actually dictate a lower R&D level for the imitator. The intuition for this finding is that the social planner is more apt than the noncooperative solution to take advantage of the aforementioned asymmetry premium for social welfare, and thus more prone to a higher dispersion in R&D levels.

Part (ii) of this result is not surprising; it simply confirms for the particular setting at hand a well known general fact about innovation in general: that the market typically undersupplies R&D, due to well-established market failures, in particular to the imperfectly appropriable nature of process R&D here. Thus, even a second-best social planner would typically choose to generate higher levels of effective R&D.

Furthermore, while total welfare achieved under symmetric regulation is inferior to that induced by the joint lab, the next result states on the contrary that asymmetric regulation often welfare-dominates the joint lab.

Proposition 7. *Total welfare induced by the asymmetric second-best welfare maximizing R&D investments satisfies the following:*

- **(i)** $W(\bar{x}_a, \underline{x}_a) > W(x_s, x_s)$,
- **(ii)** $W(\bar{x}_a, \underline{x}_a) > W_J$ *if either* $\beta \ge \frac{\sqrt{2}}{2}$ *or* $\beta < \frac{\sqrt{2}}{2}$ *and* $9\gamma < Z_3$, *where*

$$Z_3 = (1-\beta)\frac{7\beta - 11 - \sqrt{193\beta^2 - 154\beta + 49}}{2\beta^2 - 1}.$$

Part (i) is an obvious statement in that it captures the premium to the social planner of having fully flexible choices in firms' R&D levels. A quantitative assessment of the welfare loss to being subject to the symmetry constraint in R&D choices is investigated in the next subsection.

An intuitive understanding for part (ii) of this result may be achieved as follows. The comparison at hand involves two issues with respect to which the two scenarios hold opposite positions: R&D duplication costs and symmetry of R&D choices. The joint lab has the advantage of avoiding R&D duplication costs but forces firms to settle for symmetric R&D levels. On the other hand, unconstrained welfare maximization faces R&D duplication costs but allows asymmetric R&D choices. Part (ii) states that the second-best asymmetric regulation (or Case P_A) welfare-dominates the joint lab (Case J) when either the spillover parameter is high enough or else when R&D is relatively less costly.

Therefore, overall the main message of this proposition is that the flexibility to choose asymmetric R&D levels often contributes substantially to social welfare. We now examine this issue in a quantitative sense.

5.3 | The welfare cost of equal treatment

We have seen that, in industries with (nonzero) one-way spillovers, the social planner always has an incentive to engage in discriminatory regulation of the two firms in order to maximize social welfare. However, in most societies, political, moral, and other fairness considerations will dictate that the social planner engage instead in equal treatment of regulated firms, in total disregard of any resulting loss of social welfare. In this subsection, we investigate the value of the welfare loss due to the equal treatment constraint, and its comparative statics as the parameters of the model vary exogenously.

The welfare loss is defined as the difference between asymmetric and symmetric optimal second-best welfare, that is,

$$L = W(\overline{x}_a, \underline{x}_a) - W\left(x_s, x_s\right).$$

Using the expressions for the two welfare levels $W(\overline{x}_a, \underline{x}_a)$ and $W(x_s, x_s) \triangleq W$ given above, one arrives upon simplification at

$$L = \frac{16\beta^2\gamma^2(a-c)^2}{(9\gamma-4)D_a}.$$

It is easy to verify that $\frac{\partial L}{\partial \beta} > 0$. This is intuitive, because it simply reflects that, being due to the nature of the spillover process, the scope for endogenous heterogeneity of firms' post-R&D costs increases with the size of spillovers.

Furthermore, as β increases from 0 to 1, it may be verified that the welfare loss L increases from 0 to $L = \frac{16\gamma^2(a-c)^2}{(9\gamma-4)(9\gamma^2-8\gamma+8)}$.

Therefore, because W is independent of β, $\frac{L}{W}$ can be as high as $\frac{16\gamma^2(a-c)^2}{(9\gamma-4)(9\gamma^2-8\gamma+8)} / \frac{4\gamma(a-c)^2}{(9\gamma-4)} = \frac{4\gamma}{9\gamma^2-8\gamma+8}$. Maximizing the latter expression with respect to γ yields a unique argmax of $\gamma^* \approx 0.943$, and a corresponding maximal value of $\frac{L}{W}$ equal to 0.446.[16]

We have just established part (ii) of the following result (part (i) follows directly from evaluating and signing $dL/d\beta$ and $dL/d\gamma$. This is easy to do, and is thus left to the reader).

Proposition 8. *The welfare loss L due to equal treatment in R&D regulation satisfies*

- **(i)** *L is increasing in β and in $(a-c)$, and decreasing in γ.*
- **(ii)** *The maximal welfare cost of equal treatment in relative terms, $\frac{L}{W}$, is 44.6%.*

This number is remarkably high, even when understood as just a conceivable upper bound on the relative size of welfare loss. Indeed, the actual loss for a particular industry will depend on the specific values of β and γ (the lower bound of this loss is clearly 0, which is easily seen to be achieved for a spillover value of $\beta = 0$, due to the symmetric solution then). This illustration clearly indicates that this ubiquitous aversion to unequal treatment can lead to quantitatively significant losses.

The dichotomy between normative and positive (or politically constrained) efficiency emerges in several different settings in the process of implementing various aspects of public policy. Different manifestations of the same fundamental issue may be seen in a number of different studies covering various areas of economics, including, for instance, Spencer and Brander (1985), Salant and Shaffer (1998, 1999), Matsuyama (2002), Basu *et al.* (2016), Yazici (2016), Acemoglu *et al.* (2017), and Chatterjee (2017), among many others.

[16] The ratio $\frac{L}{W}$ turns out (when evaluated at $\beta = 1$) to be strictly quasi-concave in γ, so that the first-order condition is necessary and sufficient for a unique argmax. In addition, Assumptions 2 and 3 are easily seen to be satisfied around $\gamma^* \approx 0.943$.

6 | CONCLUSION

This paper has investigated the properties of a symmetric two-period R&D model that departs from the standard setting by adopting a deterministic one-way spillover structure. The latter is a reflection of the one-dimensional nature of the R&D process. Though firms are *ex ante* identical, one obtains a unique pair of asymmetric equilibria in terms of R&D investments. Thus, the roles of R&D innovator and imitator are endogenously determined as a direct consequence of the one-way spillover structure. This establishes a simple link between the nature of the R&D process in an industry—including the associated spillover—and the emergence of interfirm heterogeneity in that industry. Another goal of the paper was to investigate the relative performance of R&D cooperation through a joint lab. We find that the innovator sometimes invests more in R&D than the joint lab, and the industry's total profit is sometimes higher than under the joint lab.

The main part of the paper provides a welfare analysis in which we examine the usual question of how distortive the noncooperative equilibrium is, in terms of propensity for R&D and equilibrium welfare. To this end, we consider a realistic second-best social planner who selects firms' R&D levels but does not control their Cournot market conduct. We also compare the performance of the joint lab as an R&D cooperation scenario with the second-best optimum. Under the constraint of symmetric treatment of the firms by the planner, the socially optimal solution yields the same R&D level as the joint lab. It follows that the latter is a practical way to realize the second-best level of R&D without direct intervention.

Finally, due to the fact that the same forces that lead to asymmetric Nash equilibrium in R&D levels also lead to asymmetric (unconstrained) social optima, we investigate in some detail the social costs (or welfare loss) of imposing the politically motivated constraint of symmetric R&D investments among firms. We find that this social cost can reach the highly significant level of 45% in relative terms.

APPENDIX: PROOFS

Proof of Lemma 1. The reaction function r as given by (2) is not continuous because, letting $x^{S1} = r_1(x^{S1})$ for $x_1 \geq x_2$ and $x^{S2} = r_1(x^{S2})$ for $x_1 \leq x_2$, one obtains

$$x^{S1} = \frac{2(a-c)(2-\beta)}{(9\gamma - 2(2-\beta))}, \quad \text{and} \quad x^{S2} = \frac{4(a-c)(1-\beta)}{(9\gamma - 4(1-\beta))}$$

with $x^{S1} > x^{S2}$. Hence, the reaction function has a downward jump, and letting $\hat{x}$ be the solution to $U(r_1(\hat{x}), \hat{x}) = L(r_1(\hat{x}), \hat{x})$, we have that

$$\hat{x} = \frac{(a-c)\left(\sqrt{1 + \frac{2\beta(4-3\beta)}{(9\gamma - 2(\beta-2)^2)}} - 1\right)}{\left(2\beta - 1 + (1-\beta)\sqrt{1 + \frac{2\beta(4-3\beta)}{(9\gamma-2(\beta-2)^2)}}\right)}. \tag{A1}$$

Furthermore, $\hat{x}$ is unique since both U and L are monotonic in x_2. U is decreasing in x_2 for all $\beta \in [0, 1]$, whereas L either increases with x_2 for $\beta > 1/2$ or decreases with x_2 slower than U. ■

Proof of Proposition 1. A lengthy but simple computation establishes that $\bar{x}, \underline{x}$ as given by (3) and (4) satisfy $\overline{x} > \hat{x}$ if $9\gamma > I_1$ and $\underline{x} < \hat{x}$ if $9\gamma > I_2$, with $\hat{x}$ as defined by (10) and

$$I_1 = (5\beta^2 - 12\beta + 8) + \sqrt{13\beta^4 - 48\beta^3 + 68\beta^2 - 48\beta + 16},$$

$$I_2 = (5\beta^2 - 12\beta + 8) + \sqrt{73\beta^4 + 224\beta^2 - 216\beta^3 - 96\beta + 16}.$$

Straightforward computations then establish that $I(\beta) > I_1$ and $I(\beta) > I_2$. Hence, if Assumptions 1 through 4 hold, the pair of PSNE $(\bar{x}, \underline{x})$ and $(\underline{x}, \bar{x})$, with $\bar{x}, \underline{x}$ as given by Equations 3 and 4 is unique. ■

Proof of Proposition 2.

(i) This is seen by inspection.

(ii) We have that

$$x_J - \bar{x} = \frac{4(a-c)}{9\gamma - 4} - \frac{2(a-c)(2-\beta)\left(3\gamma - 4(\beta-1)^2\right)}{D_N}.$$

Simplifying and rearranging then leads to $sign(x_J - \bar{x}) = sign(28\beta + 9\gamma - 12\beta^2 - 16) < 0$ if and only if $9\gamma < 4(1-\beta)(4-3\beta)$.

(iii) Similarly, it can be shown that

$$x_J - \left(\underline{x} + \beta\left(\bar{x} - \underline{x}\right)\right) = \frac{4(a-c)}{9\gamma-4} - \frac{2(a-c)\left(3\left(\beta^2 - 2\beta + 2\right)\gamma - 4(2-\beta)(\beta-1)^2\right)}{27\gamma^2 - 6\gamma\left(5\beta^2 - 12\beta + 8\right) + 8(2-\beta)(1-\beta)^2}$$

$$> 0 \quad \text{if } 9\gamma > \frac{4(1-\beta)(5-3\beta)}{(2-\beta)},$$

which can be shown to hold for all the parameter values for which the Nash equilibrium is interior, or if $I(\beta) > \frac{4(1-\beta)(5-3\beta)}{(2-\beta)}$.

(iv) Total cost reductions achieved under cooperation through a joint lab formation dominate those of the noncooperative regime if

$$\frac{8(a-c)}{9\gamma-4} > (1+\beta)\frac{2(a-c)(2-\beta)\left(3\gamma - 4(\beta-1)^2\right)}{D_N} + (1-\beta)\frac{4(a-c)(1-\beta)(3\gamma - 2(\beta-1)(\beta-2))}{D_N}$$

$$\Leftrightarrow 9\gamma > \frac{12(1-\beta)(3-2\beta)}{(3-\beta)},$$

which holds at an interior equilibrium because $I(\beta) > \frac{12(1-\beta)(3-2\beta)}{(3-\beta)}$. ■

Proof of Proposition 3. We first check that $2\tilde{F}(x_J) > F(\bar{x}_B, \underline{x}_B) + F(\underline{x}_B, \bar{x}_B)$ holds for $4\frac{a}{c} < 9\gamma \leq I(\beta)$. It may be verified that the difference $2\tilde{F}(x_J) - F(\bar{x}_B, \underline{x}_B) - F(\underline{x}_B, \bar{x}_B)$ is positive if and only if

$$\begin{aligned}
&-729c^2\gamma^4 + 162c\gamma^3(13c\beta^2 + (2a-26c)\beta + 2a + 13c)\\
&-72((21 - 32\beta + 16\beta^2)(1-\beta)^2c^2 - 2a(1-\beta)(-6 + 6\beta + \beta^2)c + 2a^2)\gamma^2\\
&+32(1-\beta)^2(8(1-\beta)^2c^2 + 2a(7-8\beta)(1-\beta)c + a^2(9 - 2\beta + \beta^2))\gamma - 128a^2(1-\beta)^4 < 0.
\end{aligned}$$

Numerical computations then establish that this inequality holds for $9\gamma \in [4\frac{a}{c}, I(\beta)]$.

In a similar fashion, we now shall show that $2\tilde{F}(c) > F(\bar{x}, \underline{x}) + F(\underline{x}, \bar{x})$ for $I(\beta) < 9\gamma < 4\frac{a}{c}$. A lengthy computation establishes that the sign of the difference $2\tilde{F}(c) - F(\bar{x}, \underline{x}) - F(\underline{x}, \bar{x})$ is the same as that of L, where

$$\begin{aligned}
L = &-6561c^2\gamma^5 + \gamma^4(14\,580c^2\beta^2 - 34\,992c^2\beta + 20\,412c^2 + 5832ac)\\
&+\gamma^3(972a^2\beta^2 - 1944a^2\beta - 14\,904ac\beta^2 + 34\,992ac\beta c^2\\
&-20\,736ac - 8100c^2\beta^4 + 42\,768c^2\beta^3 - 80\,676c^2\beta^2 + 64\,152c^2\beta - 18\,144)\\
&+\gamma^2(9072a^2\beta^3 - 2232a^2\beta^4 - 11\,592a^2\beta^2 + 4320a^2\beta + 576a^2 + 11\,664ac\beta^4 - 56\,160ac\beta^3 + 101\,520ac\beta^2\\
&-81\,216ac\beta + 24\,192ac - 4320c^2\beta^5 + 21\,816c^2\beta^4 - 41\,904c^2\beta^3 + 37\,368c^2\beta^2 - 14\,688c^2\beta + 1728c^2)\\
&+\gamma(1152a^2\beta^6 - 7296a^2\beta^5 + 17\,664a^2\beta^4 - 19\,584a^2\beta^3 + 8064a^2\beta^2 + 1536a^2\beta - 1536a^2 - 2304ac\beta^6\\
&+18\,432ac\beta^5 - 59\,904ac\beta^4 + 101\,376ac\beta^3 - 94\,464ac\beta^2 + 46\,080ac\beta - 9216ac + 576c^2\beta^6 - 4608c^2\beta^5
\end{aligned}$$

$$+ 14\,976c^2\beta^4 - 25\,344c^2\beta^3 + 23\,616c^2\beta^2 - 11\,520c^2\beta + 2304 + c^2)$$

$$+ (256a^2\beta^6 - 2048a^2\beta^5 + 6656a^2\beta^4 - 11\,264a^2\beta^3 + 10\,496a^2\beta^2 - 5120a^2\beta + 1024a^2).$$

For apparent reasons, we had to rely on numerical computations then to demonstrate that $L > 0$ for $9\gamma \in [I(\beta), 4\frac{a}{c}]$. ■

Proof of Proposition 5. We first shall show that $W_J > W(x_W, x_W)$. We have that

$$4\frac{(9\gamma-2)(a-c)^2\gamma}{(9\gamma-4)^2} > 4\frac{(a-c)^2\gamma}{(9\gamma-4)} \Leftrightarrow 9\gamma > 3,$$

which holds from Assumption 2 and the fact that $9\gamma > 4\frac{a}{c}$.

Next, we establish that $W_J \geq W(\bar{x}, \underline{x})$. From Section 3.5, we have that $\frac{1}{2}F(x_J) > F(\underline{x}, \bar{x})$ and $\frac{1}{2}F(x_J) > F(\bar{x}, \underline{x})$ for $9\gamma > \max\{4\frac{a}{c}, I(\beta)\}$. Hence, it directly follows that the industry's profit under the joint lab formation exceeds that of the noncooperative setting, that is,

$$F(x_J) > F(\underline{x}, \bar{x}) + F(\bar{x}, \underline{x}). \tag{A2}$$

Therefore, a sufficient condition for $W_J \geq W(\bar{x}, \underline{x})$ to hold is that consumer surplus when firms cooperate through a joint lab is higher. The difference $CS(x_J, x_J) - CS(\bar{x}, \underline{x})$ is given by

$$2\left(\frac{3\gamma(a-c)}{(9\gamma-4)}\right)^2 - \frac{18\,(3\gamma+(1-\beta)(3\beta-4))^2\,(a-c)^2\gamma^2}{\left(27\gamma^2-6\gamma\left(5\beta^2-12\beta+8\right)-8(\beta-2)(\beta-1)^2\right)^2}.$$

Straightforward computations then establish that $CS(x_J, x_J) - CS(\bar{x}, \underline{x}) \geq 0$ if and only if $K_1.K_2 \geq 0$, where

$$K_1 = (4(1-\beta)(2\beta^2-9\beta+8) + 54\gamma^2 - 3\gamma(19\beta^2-45\beta+32)),$$

$$K_2 = (3\gamma(3-\beta) - 4(1-\beta)(3-2\beta)).$$

Both K_1 and K_2 are positive for all $9\gamma > I(\beta)$. Thus, we have that

$$CS\left(x_J, x_J\right) \geq CS(\bar{x}, \underline{x}). \tag{A3}$$

Hence, (11) together with (12) establish the superiority of the joint lab in terms of welfare.

Finally, the difference $W(\bar{x}, \underline{x}) - W(x_W, x_W)$ is given by

$$\frac{2\gamma(a-c)^2\left[162\gamma^3 - 9\gamma^2\left(41\beta^2-96\beta+56\right) + 3\gamma\left(81\beta^2-224\beta+160\right)(1-\beta)^2 - 32(2-\beta)^2(1-\beta)^4\right]}{\left(27\gamma^2-6\gamma\left(5\beta^2-12\beta+8\right)-8(\beta-2)(\beta-1)^2\right)^2}$$

$$-4\frac{(9\gamma-2)(a-c)^2\gamma}{(9\gamma-4)^2}.$$

Simplifying and rearranging, we have that

$$W(\bar{x}, \underline{x}) > W\left(x_W, x_W\right) \Leftrightarrow 9\gamma \in \left(K_3, K\right),$$

with K as indicated in the proposition and

$$K_3 = \frac{1}{2}(43\beta^2 - 102\beta + 55) - \frac{1}{2}\sqrt{1057 - 4212\beta + 5870\beta^2 - 3396\beta^3 + 697\beta^4}.$$

It can be checked that $K_3 < I(\beta)$. ■

Proof of Proposition 6.

(i) Upon simplification, the sign of the difference $\bar{x}_W - \bar{x}$ is the same as that of $8(16-11\beta)(1-\beta)^3 + 81\gamma^2 - 18\gamma(1-\beta)(11-9\beta)$, which is strictly positive for $9\gamma > 18(1+\beta)$.

Likewise, it may be easily verified that the sign of $\underline{x}_W - \underline{x}$ is the same as that of $9\gamma(3\beta-2) - 2(1-\beta)(23\beta - 11 - 11\beta^2)$. This expression is strictly positive if both $\beta > \frac{2}{3}$ and $9\gamma > 18(1+\beta)$, so that $\underline{x}_W > \underline{x}$. Instead, if either $\beta = \frac{2}{3}$, or $\beta < \frac{2}{3}$ and $9\gamma > \frac{2(1-\beta)(23\beta-11-11\beta^2)}{(3\beta-2)}$, then $\underline{x}_W < \underline{x}$.

(ii) As for total effective cost reductions, straightforward computations establish that $(1+\beta)\bar{x} + (1-\beta)\underline{x} - (1+\beta)x_1^W - (1-\beta)\underline{x}_W < 0$ if $-9(1+\beta)\gamma^2 - 2(1-\beta)(-15+8\beta+3\beta^2)\gamma + 8(2\beta-3)(1-\beta)^3 < 0$, which holds for any $\beta \in [0,1]$ and $9\gamma > Z_1$. ■

Proof of Proposition 7.

(i) We have that

$$\begin{aligned} W(\bar{x}_a, \underline{x}_a) - W(x_s, x_s) &= \frac{4(\gamma - 2(1-\beta)^2)(a-c)^2\gamma}{9\gamma^2 - 2\gamma(11\beta^2 - 18\beta + 11) + 8(1-\beta)^2} - 4\frac{(a-c)^2\gamma}{(9\gamma-4)} \\ &= \frac{16(a-c)^2\beta^2\gamma^2}{(9\gamma-4)(9\gamma^2 - 2\gamma\left(11\beta^2 - 18\beta + 11\right) + 8(1-\beta)^2)} \\ &> 0. \end{aligned}$$

(ii) The difference $W(\bar{x}_a, \underline{x}_a) - W_J$ is given by

$$\frac{4\left(\gamma - 2(1-\beta)^2\right)(a-c)^2\gamma}{9\gamma^2 - 2\gamma\left(11\beta^2 - 18\beta + 11\right) + 8(1-\beta)^2} - 4\frac{(9\gamma-2)(a-c)^2\gamma}{(9\gamma-4)^2}$$

so that $W(\bar{x}_a, \underline{x}_a) - W_J > 0$ if $16(1-\beta)^2 + 18\gamma^2(1-2\beta^2) - 4\gamma(1-\beta)(11-7\beta) > 0$, which holds if either $\beta < \frac{1}{2}\sqrt{2}$ and $9\gamma < Z_2$, or $\beta \geq \frac{1}{2}\sqrt{2}$ provided that $9\gamma > 18(1+\beta)$. ■

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How to cite this article: Cosandier C, Feo GD, Knauff M. Equal treatment and socially optimal R&D in duopoly with one-way spillovers. *J Public Econ Theory*. 2017; 00:1–19. https://doi.org/10.1111/jpet.12280